EFb — Type Inference

$\tau ::= Bool \mid Int \mid \tau \to \tau \mid \alpha$

$E ::= \{\tau = \tau, \ldots\}$

$\alpha ::= 'a \mid 'b \mid \cdots$

$$\boxed{\Gamma \vdash e : \tau \setminus E}$$

1. Construct a type for expression via $\Gamma \vdash e : \tau \setminus E$.
2. Perform deductive closure.
3. Check for consistency.
4. Type substitution

$$\text{Var}\ \frac{(x:\tau) \in \Gamma}{\Gamma \vdash x : \tau \setminus \emptyset}$$

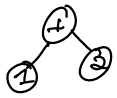
$$\text{Not}\ \frac{\Gamma \vdash e : \tau \setminus E}{\Gamma \vdash \text{Not}\ e : Bool \setminus E \cup \{\tau = Bool\}}$$

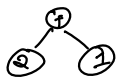
In TFb, there is only one way to write each type.
In EFb, the "same" type can be written in different ways.

$$\text{Function}\ \frac{\Gamma, x:\alpha \vdash e : \tau \setminus E \qquad \alpha \text{ is fresh}}{\Gamma \vdash \text{Function}\ x \to e : \alpha \to \tau \setminus E}$$

Are these the same?

1+2 2+1





$Bool \setminus \emptyset$ $\qquad$ $'a \setminus \{'a = Bool\}$

$'a \setminus \{'a = 'b,\ 'b = Bool\}$

$'a \to Bool \setminus \{'a = Bool\}$

$$\text{Not}\ \frac{\text{Int}\ \dfrac{}{\emptyset \vdash 4 : Int \setminus \emptyset}}{\emptyset \vdash \text{Not}\ 4 : Bool \setminus \{Int = Bool\}}$$

Deductive Closure

Transitivity 1. If $\tau_1 = \tau_2$ and $\tau_2 = \tau_3$ then $\tau_1 = \tau_3$.

Symmetry 2. If $\tau_1 = \tau_2$ then $\tau_2 = \tau_1$.

Function 3. If $\tau_1 \to \tau_2 = \tau_3 \to \tau_4$ then $\tau_1 = \tau_3$ and $\tau_2 = \tau_4$.

match foo with
| B b → f b
| A a → a + 1
| C → 0

$'a \setminus \{'a = Bool,\ 'b = 'a\}$ $\qquad$ $'b = Bool,\ Bool = 'a,\ 'a = 'b,\ Bool = 'b$ (trans., sym.)

Perform deductive closure: apply every rule as much as possible, even to new equations.

$\{'a \to 'b = 'b \to Bool\}$ $\qquad$ $'a = 'b,\ 'b = Bool,\ 'b \to Bool = 'a \to 'b,\ 'a = Bool,\ 'b = 'a,\ Bool = 'a,\ Bool = Bool,\ 'a = 'a,\ 'b = 'b$

$\{Int = 'a,\ 'a = Bool\}$ $\qquad$ $'a = Int,\ Bool = 'a,\ Int = Bool,\ Bool = Int$

Consistency Checking

Does the set equate different kinds of types?

$Int = \tau \to \tau'$
$Int = Bool$
$Bool = \tau \to \tau'$

Type Substitution

$'a \to Int \setminus \{ 'a = 'b \to Bool,\ 'b = Bool \}$

have $'a \to Int \setminus \{ 'a = 'b \to Bool,\ 'b = Bool,\ Bool = 'b,\ 'b \to Bool = 'a,\ 'b \to Bool = 'b \to Bool,\ 'b = 'b,\ Bool = Bool \}$

want $(Bool \to Bool) \to Int$

$('a) \to Int$

$('b \to Bool) \to Int$

$(Bool \to Bool) \to Int$

- Find a type variable α in type
- Find an eqn $\alpha = \tau$
- Replace α with τ

Repeat until we can't replace

When picking an equation, order on τ.

1. Concrete types are smallest.
2. Type variables are sorted by name.

Pick "smallest" type; never pick a type variable $\geq$ yourself.

$'b \to 'a \setminus \{ 'a = 'b,\ 'b = 'a,\ 'a = 'a,\ 'b = 'b \}$

$('b) \to 'a$

$'a \to 'a$

$'b \to 'a$

fun x -> failwith "

$'a \to 'b \setminus \{ 'b = 'c,\ 'c = Bool,\ 'a = 'b, \ldots \}$

type variable ↓ type variable

Function a -> If a Then H = 3 Else a

Perform 4 steps (as far as you can)

EIFb cannot typecheck self-passing.

$'a \setminus \{ 'a \to 'b = 'a \}$

$('a)$

$('a) \to 'b$

$(('a) \to 'b) \to 'b$

$((('a \to 'b) \to 'b) \to 'b$