TFbR

Evaluation Relations $e \Rightarrow v$

Typing Relation $\Gamma \vdash e : \tau$

$$(\text{Function } a:\{q:\text{Int}\} \to \overbrace{a.q}^{\text{destruct record}})\ \overbrace{\{q=5\}}^{\text{construct record}} \Rightarrow 5$$

$$\text{Appl}\ \cfrac{\text{Fun}\ \cfrac{\text{Proj}\ \cfrac{\text{Var}\ \cfrac{}{\{a:\{q:\text{Int}\}\} \vdash a : \{q:\text{Int}\}}}{\{a:\{q:\text{Int}\}\} \vdash a.q : \text{Int}}}{\emptyset \vdash (\text{Function } a:\{q:\text{Int}\} \to a.q) : \{q:\text{Int}\} \to \text{Int}} \qquad \text{Rec}\ \cfrac{\text{Int}\ \cfrac{}{\emptyset \vdash 5 : \text{Int}}}{\emptyset \vdash \{q=5\} : \{q:\text{Int}\}}}{\emptyset \vdash (\text{Function } a:\{q:\text{Int}\} \to a.q)\ \{q=5\} : \text{Int}}$$

$$\text{Appl}\ \cfrac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'}$$

$$\text{Fun}\ \cfrac{\Gamma, x:\tau \vdash e : \tau'}{\Gamma \vdash (\text{Function } x:\tau \to e) : \tau \to \tau'}$$

$$\text{Proj}\ \cfrac{\Gamma \vdash e : \{l_1:\tau_1, \ldots, l_n:\tau_n\} \quad l = l_k}{\Gamma \vdash e.l : \tau_k}$$

$$\text{Rec}\ \cfrac{\Gamma \vdash e_1 : \tau_1 \quad \cdots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash \{l_1 = e_1, \ldots, l_n = e_n\} : \{l_1:\tau_1, \ldots, l_n:\tau_n\}}$$

FbR

$$(\text{Function } a \to a.q)\ \{q=5\} \Rightarrow 5$$

$$(\text{Function } a \to a.q)\ \{q=5, z=\text{True}\} \Rightarrow 5$$

TFbR

w.t.s. $\emptyset \vdash (\text{Function } a : \{q:\text{Int}\} \to a.q)\ \underset{\substack{\uparrow \\ \text{has type } \{q:\text{Int},\ z:\text{Bool}\}}}{\{q=5, z=\text{True}\}} : \text{Int}$ $\longleftarrow$ This claim does not hold

Don't match

This is disappointing b/c the record I have contains everything I need <u>and more</u>!

Subtyping

"A type is a set of values" *

"A subtype is a subset of values" *

Eval $e \Rightarrow v$

Typing $\Gamma \vdash e : \tau$

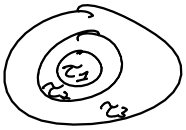
Subtyping $\tau <: \tau$

Grammar: $\tau <: \tau$ $\boxed{<:}$ $\leq$ $\subseteq$

Meaning: $\tau_1 <: \tau_2$ claims that all values in τ_1 are also in τ_2

Intuition: If $\tau_1 <: \tau_2$ then τ_1 can be used anywhere that τ_2 is expected.

TFbR



Reflexivity $$\frac{}{\tau <: \tau}$$

Transitivity $$\frac{\tau_1 <: \tau_2 \qquad \tau_2 <: \tau_3}{\tau_1 <: \tau_3}$$

~~Symmetry $\frac{\tau_1 <: \tau_2}{\tau_2 <: \tau_1}$~~

Record $$\frac{\tau_1 <: \tau_1' \quad \cdots \quad \tau_n <: \tau_n'}{\{l_1 : \tau_1, \ldots l_n : \tau_n, \ldots, l_m : \tau_m\} <: \{l_1 : \tau_1', \ldots, l_n : \tau_n'\}}$$

$\text{Int} <: \text{Int}$

$\{q : \text{Int}, z : \text{Bool}\} <: \{q : \text{Int}\}$

$\{q : \text{Int}, y : \text{Int}, z : \text{Bool}\} <: \{q : \text{Int}, z : \text{Bool}\}$

Write a record rule

$\{d : \{q : \text{Int}, z : \text{Bool}\}\} <: \{d : \{q : \text{Int}\}\}$

Function $r : \{d : \{q : \text{Int}\}\} \to r.d.q$

More fields: "width subtyping"

More specific fields: "depth subtyping"

STFbR

$$\frac{\Gamma \vdash e : \tau \qquad \tau <: \tau'}{\Gamma \vdash e : \tau'}$$

$$\frac{\dfrac{\dfrac{}{\emptyset \vdash 5 : \text{Int}} \qquad \dfrac{}{\emptyset \vdash \text{False} : \text{Bool}}}{\emptyset \vdash \{a = 5, b = \text{False}\} : \{a : \text{Int}, b : \text{Bool}\}} \qquad \dfrac{\dfrac{}{\text{Int} <: \text{Int}}}{\{a : \text{Int}, b : \text{Bool}\} <: \{a : \text{Int}\}}}{\emptyset \vdash \{a = 5, b = \text{False}\} : \{a : \text{Int}\}}$$

Subtyping Functions

STFbR $\tau ::= \text{Int} \mid \text{Bool} \mid \{l:\tau,\dots\} \mid \tau \to \tau$

$$(\text{Function } f: \{a:\text{Int}\} \to \{b:\text{Int}\} \to f\ \{a=6\})$$

$$\underbrace{(\text{ Function } r:\{a:\text{Int}\} \to \{b=r.a,\ c=\text{False}\})}_{\{a:\text{Int}\} \longrightarrow \{b:\text{Int},\ c:\text{Bool}\}}$$

Focusing on output:

(Not the whole picture)

$$\frac{\tau_1 <: \tau_2}{\tau \to \tau_1 <: \tau \to \tau_2}$$

With input (the whole picture)

Func Sub $$\frac{\tau_2' <: \tau_1' \quad \tau_1 <: \tau_2}{\tau_1' \to \tau_1 <: \tau_2' \to \tau_2}$$

"contravariance on input"