Encodings vs. Definitions

Let

$$\frac{e_1 \Rightarrow v_1 \qquad e_2[v_1/x] \Rightarrow v_2}{\text{Let } x = e_1 \text{ In } e_2 \Rightarrow v_2}$$

] Definition

$$\text{Let } x = e_1 \text{ In } e_2 \overset{\text{def}}{=\!=} (\text{Function } x \to e_2)\ e_1$$

] Encoding

<u>Defining Pairs</u> } · Fb values

FbP

$$V ::= \underbrace{\cdots\cdots}_{} \mid (v, v)$$

$$e ::= \underbrace{\cdots\cdots}_{Fb\ expr} \mid (e, e) \mid Fst\ e \mid Snd\ e$$

$$\overbrace{(1+1, 2)}^{e} \Rightarrow \overbrace{(2,2)}^{v}$$

Syntax

Let $e \Rightarrow v$ be the relation defined by Fb $e \Rightarrow v$ rules | Semantics
as well as the following rules:

$$\frac{e_1 \Rightarrow v_1 \qquad e_2 \Rightarrow v_2}{(e_1, e_2) \Rightarrow (v_1, v_2)}$$

$$\frac{e \Rightarrow (v_1, v_2)}{Fst\ e \Rightarrow v_1}$$

$$Fst\ (1,\ True+1) \neq$$

$$\frac{e \Rightarrow (v_1, v_2)}{Snd\ e \Rightarrow v_2}$$

FbP Encoding in Fb

FbP exprs $\longrightarrow$ Fb exprs

First draft: ~~$(e_1, e_2) \stackrel{def}{=}$ Function b $\rightarrow$ If b Then e_1 Else e_2~~

$(e_1, e_2) \stackrel{def}{=}$ Let $\overset{*}{x} = e_1$ In Let $\overset{*}{x}' = e_2$ In Function b $\rightarrow$ If b Then $\overset{*}{x}$ Else $\overset{*}{x}'$

Fst e $\stackrel{def}{=}$ e True

Snd e $\stackrel{def}{=}$ e False

Fst (Function a $\rightarrow$ Not a)

Encoding

- Confusion between encoded form & other code
+ Works w/ existing lang

Defn

+ Real representation
- Requires new features & rules

type expr =
  :
  :
  '

[@@deriving eq, ord, show]
  ;;

Encoding FbLP into FbP

$$e ::= \; \cdots \; | \; \text{Nil} \; | \; \text{Cons} \; e \; e \; | \; \text{Hd} \; e \; | \; \text{Tl} \; e \; | \; \text{IsEmpty} \; e$$

$$\boxed{v ::= \; \cdots \; | \; \text{Nil} \; | \; \text{Cons} \; v \; v}$$

$$\underset{(\text{True},-)}{\uparrow} \quad \underset{(\text{False}, (\text{head},\text{tail}))}{\uparrow}$$

$$\frac{e \Longrightarrow (v_1, v_2)}{\text{IsEmpty} \; e \Longrightarrow v_1}$$

$$\text{Nil} \;\overset{\text{def}}{=}\; (\text{True}, 0)$$

$$\text{Cons} \; e_1 \; e_2 \;\overset{\text{def}}{=}\; (\text{False}, (e_1, e_2))$$

$$\text{IsEmpty} \; e \;\overset{\text{def}}{=}\; \text{Fst} \; e$$

$$\text{Hd} \; e \;\overset{\text{def}}{=}\; \text{Fst} \; (\text{Snd} \; e)$$

$$\text{Tl} \; e \;\overset{\text{def}}{=}\; \text{Snd} \; (\text{Snd} \; e)$$