If - Then - Else

$e \Rightarrow v$

If True

$$\frac{e_1 \Rightarrow \text{True} \qquad e_2 \Rightarrow v}{\text{If } e_1 \text{ Then } e_2 \text{ Else } e_3 \Rightarrow v}$$

If False

$$\frac{e_1 \Rightarrow \text{False} \qquad e_3 \Rightarrow v}{\text{If } e_1 \text{ Then } e_2 \text{ Else } e_3 \Rightarrow v}$$

Bad If Rule (problem)

$$\chi \quad \frac{e_1 \Rightarrow v_1 \qquad e_2 \Rightarrow v_2 \qquad e_3 \Rightarrow v_3}{\text{If } e_1 \text{ Then } e_2 \text{ Else } e_3 \Rightarrow \begin{cases} v_2 & \text{when } v_1 = \text{True} \\ v_3 & \text{when } v_1 = \text{False} \end{cases}}$$

If True
Then 4
Else 1+False

Calling Functions

(Function a → a + 1) 5 ⟹ 6

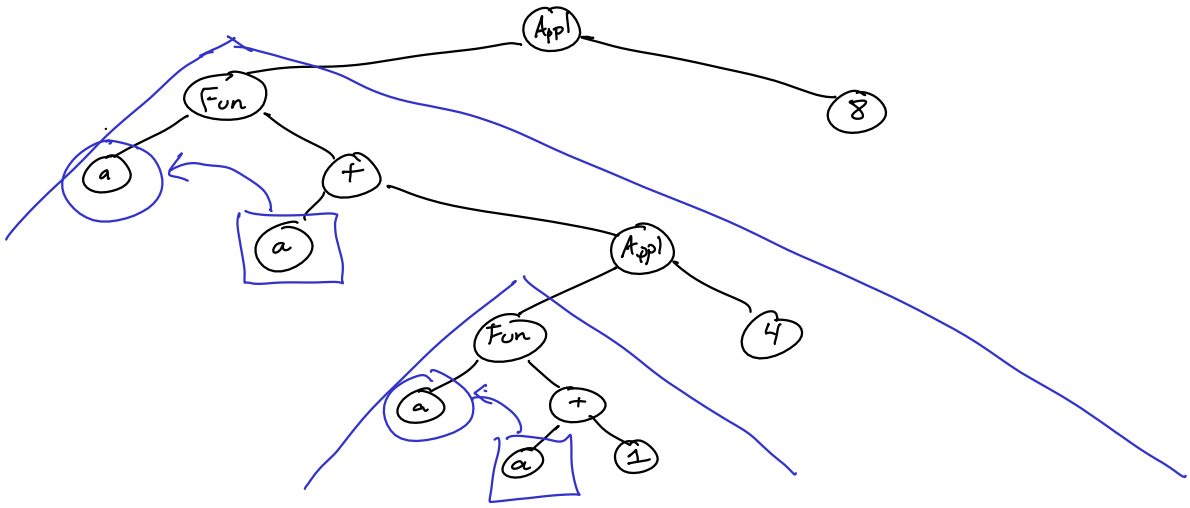
b/c 5 + 1 ⟹ 6

replacement ↓ ↙ being replaced

$e[v/x]$

(Function b → Function x → If b Then x Else 0)

(Function a → a + (Function a → a + 1) 4) 8



$4[v/x] = 4$

$(e_1 + e_2)[v/x] = (e_1[v/x]) + (e_2[v/x])$

$(\text{Function } x' \to e)[v/x] = (\text{Function } x' \to e[v/x]) \quad x' \neq x$

$(\text{Function } x \to e)[v/x] = \text{Function } x \to e$

(Function a → a + 1) 5

$(a + 1)[5/a]$

Application Rule

$$\frac{e_1 \Rightarrow \text{Function } x \to e_3 \quad e_2 \Rightarrow v_2 \quad e_3[v_2/x] \Rightarrow v_3}{e_1\ e_2 \Longrightarrow v_3}$$

e_1 e_2

(Function a → a + 1) 5

$(a+1)[5/a] = (a[5/a]) + (1[5/a])$

$= 5 + 1 \Longrightarrow 6$

Let Rule

$$\frac{e_1 \Rightarrow v_1 \qquad e_2[v_1/x] \Rightarrow v_2}{\text{Let } x = e_1 \text{ In } e_2 \Longrightarrow v_2}$$

Let Rec in F1

fn param fn & param fn

e ::= ... | Let Rec x x = e In e

CHECK BOOK

$e_2[(\text{Function } x_2 \to e_1[\text{Let Rec } x_1\ x_2 = e_1 \text{ In } x_1 / x_1])/x_1] \Longrightarrow v_2$

Let Rec x_1 x_2 = e_1 In e_2 $\Longrightarrow$ v_2

Self-passing recursion

```
Let sum = Function a ->
            If a=0 Then 0 Else a+sum (a-1)
In
sum 2
```

2+sum (2-1)

```
Let sum0 = Function self -> Function a ->
             If a=0 Then 0 Else a + self self (a-1)
In
sum0 sum0 2
```

```
(Function self -> Function a ->
     If a=0 Then 0 Else a + self self (a-1))
(Function self -> Function a ->
     If a=0 Then 0 Else a + self self (a-1))
2
```

```
(Function a -> If a=0 Then 0 Else a +
    (Function self -> Function a ->
         If a=0 Then 0 Else a + self self (a-1))
    (Function self -> Function a ->
         If a=0 Then 0 Else a + self self (a-1)) (a-1))
2
```