Logic

$\forall$ for all

$\exists$ there exists

It is cloudy.

I am flying.

$\forall day \in days.\ cloudy(day)$

$\underbrace{foo}_{\text{predicate}}(bar)$

$P(n)$

Quantifier Ordering

$\exists x. \forall y.\ x > y$ there is a number which is bigger than every number FALSE

$\forall y. \exists x.\ x > y$ for each number, some number is bigger than it TRUE

$\forall n \in S.\ P(n)$

$\forall n.\ n \in S \wedge P(n)$ $\vee$

and or

If I have no work and if is the weekend then I will sleep in.

if work $= \emptyset \ \wedge$ weekend(today) then sleepIn(me)

$$\text{work} = \emptyset \wedge \text{weekend}(\text{today}) \Longrightarrow \text{sleepIn}(\text{me})$$

not going to use this for implication*

$$\frac{\text{work} = \emptyset \qquad \text{weekend}(\text{today})}{\text{sleepIn}(\text{me})}$$

Strings composed of $\triangle$ $\square$ ϟ

Inference Rules:

axiom (no premises)

$$1\ \frac{\text{ϟ}}{\text{ϟ}\square} \qquad 2\ \frac{\square}{\square\square} \qquad 3\ \frac{\text{ϟ}\square}{\text{ϟ}\triangle} \qquad 4\ \frac{\text{ϟ}\square}{\text{ϟ}\triangle\square} \qquad 5\ \frac{\text{ϟ}\triangle}{\triangle\triangle} \qquad 6\ \frac{}{\text{ϟ}}$$

Prove $\triangle\triangle$

ϟ therefore ϟ$\square$; therefore ϟ$\triangle$ therefore $\triangle\triangle$

$$5\ \cfrac{3\ \cfrac{1\ \cfrac{6\ \cfrac{}{\text{ϟ}}}{\text{ϟ}\square}}{\text{ϟ}\triangle}}{\triangle\triangle}$$

Metavariable.

s ::= strings containing $\lightning$, $\triangle$, $\Box$ and nothing else

$$1\ \frac{\lightning}{\lightning\Box\Box} \qquad 2\ \frac{s\Box}{s\triangle} \qquad 3\ \frac{s\Box}{s\triangle\triangle} \qquad 4\ \frac{\lightning s}{\Box s} \qquad 5\ \frac{\Box s \triangle}{s} \qquad 6\ \frac{}{\lightning}$$

Prove "".

Prove "$\Box$".

$$s=\epsilon\ 5\ \dfrac{s=\text{“}\Box\triangle\text{”}\ 5\ \dfrac{s=\text{“}\Box\triangle\triangle\text{”}\ 4\ \dfrac{s=\text{“}\lightning\Box\text{”}\ 3\ \dfrac{1\ \dfrac{6\ \dfrac{}{\lightning}}{\lightning\Box\Box}}{\lightning\Box\triangle\triangle}}{\Box\Box\triangle\triangle}}{\Box\triangle}}{\epsilon}$$

$$s=\epsilon\ 4\ \dfrac{6\ \dfrac{}{\lightning}}{\Box}$$

$$s=\text{“}\Box\text{”}\ 5\ \dfrac{s=\text{“}\Box\Box\text{”}\ 2\ \dfrac{s=\text{“}\Box\Box\text{”}\ 4\ \dfrac{1\ \dfrac{6\ \dfrac{}{\lightning}}{\lightning\Box\Box}}{\Box\Box\Box}}{\Box\Box\triangle}}{\Box}$$

$$1\ \frac{s}{s\square\square} \qquad 2\ \frac{s_1 \text{ϟ} s_2}{s_1 \triangle \text{ϟ} s_2} \qquad 3\ \frac{s_1 \square \triangle s_2}{s_1 s_2} \qquad 4\ \frac{s_1 \qquad s_2}{s_1 \text{ϟ} s_2} \qquad 5\ \frac{}{\text{ϟ}}$$

Prove: $\triangle\triangle\text{ϟϟϟ}\square\square$

$$2\ \cfrac{2\ \cfrac{1\ \cfrac{4\ \cfrac{5\ \cfrac{}{\text{ϟ}} \qquad 5\ \cfrac{}{\text{ϟ}}}{\text{ϟϟϟ}}}{\text{ϟϟϟ}\square\square}}{\triangle\text{ϟϟϟ}\square\square}}{\triangle\triangle\text{ϟϟϟ}\square\square}$$

$$4\ \cfrac{2\ \cfrac{2\ \cfrac{5\ \cfrac{}{\text{ϟ}}}{\triangle\text{ϟ}}}{\triangle\triangle\text{ϟ}} \qquad 1\ \cfrac{5\ \cfrac{}{\text{ϟ}}}{\text{ϟ}\square\square}}{\triangle\triangle\text{ϟϟϟ}\square\square}$$

Relations

$R \subseteq S \times T$

S: strings

T: $\mathbb{Z}$

len $\{(\text{""}, 0), (\text{"a"}, 1), \ldots\}$

Relation $\subseteq \mathbb{Z} \times \mathbb{Z}$

"$\leq$"

$$\text{"}\leq\text{"} = \{(2,4), (2,5), \ldots$$

$$\frac{n \in \mathbb{Z}}{n \leq n} \qquad \frac{n \leq m}{n \leq m+1} \qquad \frac{n \leq m}{n-1 \leq m}$$