TFb

If $e \Rightarrow v$ and $\Gamma \vdash e : \tau$ then $\Gamma \vdash v : \tau$

By induction on the height of proof of $e \Rightarrow v$, then by case analysis.

Value Rule case: know $e \Rightarrow v$ uses Value Rule, so $e = v$.

If $\Gamma \vdash e : \tau$ uses the Int Rule, then $e = n$ and so $\Gamma \vdash n : Int$ by Int Rule.
If $\Gamma \vdash e : \tau$ uses the Bool Rule, then ..... .
If $\Gamma \vdash e : \tau$ uses the Function Rule, then since $e = v$,
$\Gamma \vdash v : \tau$ by Function Rule.

Plus Rule case: know $e \Rightarrow v$ uses Plus Rule so $e = e_1 + e_2$
also know $e_1 \Rightarrow v_1$, $e_2 \Rightarrow v_2$, $v_1, v_2 \in \mathbb{Z}$, $v \in \mathbb{Z}$
~~also by + Rule $\Gamma \vdash e_1 : Int$, $\Gamma \vdash e_2 : Int$~~
~~so by induction, $\Gamma \vdash v_1 : Int$, $\Gamma \vdash v_2 : Int$~~
$\Gamma \vdash v : Int$ by Int Rule.

If True: (Induction, case analysis)

$e = If\ e_1\ Then\ e_2\ Else\ e_3$ $\quad \Gamma \vdash e_1 : Bool$

$e_1 \Rightarrow True$
$e_2 \Rightarrow v$ $\quad$ by ind $\quad$ $\Gamma \vdash e_2 : \tau$

$\Gamma \vdash e_3 : \tau$

$\Gamma \vdash v : \tau$.

Application Case:

$e = e_1\ e_2$
$e_1 \Rightarrow Function\ x:\tau' \to e'$
$e_2 \Rightarrow v'$
$\Gamma \vdash e_1 : \tau_1 \to \tau_2$
$\Gamma \vdash e_2 : \tau_1$
$\Gamma \vdash v' : \tau_1$ by ind
$\Gamma \vdash (Function\ x:\tau' \to e') : \tau_1 \to \tau_2$ by ind
$\Gamma, x:\tau_1 \vdash e' : \tau_2$
$\Gamma \vdash e'[v'/x] : \tau_2$ by substitution lemma
$e'[v'/x] \Rightarrow v$ by ind
$\Gamma \vdash v : \tau_2$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\ e_2 : \tau_2}$$

$$\frac{e_1 \Rightarrow Function\ x:\tau' \to e' \quad e_2 \Rightarrow v' \quad e'[v'/x] \Rightarrow v}{e_1\ e_2 \Rightarrow v}$$

$\tau' = \tau_1$

$$\frac{\Gamma, x:\tau_1 \vdash e' : \tau_2}{\Gamma \vdash (Function\ x:\tau_1 \to e') : \tau_1 \to \tau_2}$$

Soundness

$\Gamma \vdash e : \tau$ and $e \Rightarrow v$ then $\Gamma \vdash v : \tau$

Dealing with stuck cases

$e = 1 + \text{True}$

$v ::= \ldots \mid \lightning$

$$\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1, v_2 \in \mathbb{N} \quad v \text{ is sum of } v_1, v_2}{e_1 + e_2 \Rightarrow v}$$

$$\frac{e_1 \Rightarrow \lightning}{e_1 + e_2 \Rightarrow \lightning}$$

$Fb^{\lightning}$

Every expression in every situation has at least one rule which applies

Fb: converge – value
diverge – stuck
diverge – inf. loop

$Fb^{\lightning}$: converge – value
converge – stuck
diverge – inf. loop