Operational Equivalence

$1+2+3$ $\quad\quad$ $1+5$

\cong

Op Eq: $e_1 \cong e_2$

means

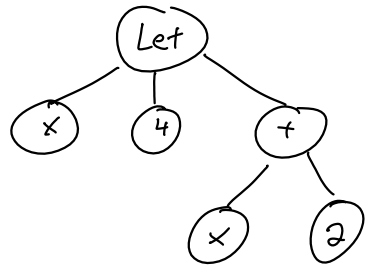
$\forall C.\ C[e_1] \Longrightarrow v$ iff $C[e_2] \Longrightarrow v'$

Context C is an expression w/ a hole $\bullet$

$\emptyset \vdash$ Let x=4 In x+2 : Int

$C =$ Let x=4 In $\bullet$ +2

$C[1+3]$ $\quad$ $C[x]$



$C[$Function y → y$] =$ Let x=4 In (Function y→y) +2

Function x → x $\quad\quad$ (Function y → Function x → x) 5

$3 \neq 4$

If $\bullet$ =3 Then True Else 1 + False

Op Eq: $e_1 \cong e_2$
iff $\forall C.\ C[e_1] \Rightarrow v$
iff
$C[e_2] \Rightarrow v'$

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1. (Function x → $\bullet$) True
2. $\bullet$ ○
3. (Function f → Not (f 4 2)) $\bullet$

1. Not x
2. Function x → x = 1
3. (Function x → Function y → x-2 = y)

$C =$ (Function x → Function y → y) $\bullet$

$C[e] \Longrightarrow$ False

1. Any e
2. e cannot contain any booleans

$C[$(Fun x → x x)(Fun x→ x x)$] \not\Rightarrow$

(Function x → Function y → y) ((Fun x →x x) (Fun x→x x))

$x \not\cong x + 1 - 1$ $\quad$ $x + 0 \cong x + 1 - 1$

(Function x → If • Then 0 Else 0) False

(Function x → y) $\not\cong$ (Function x → z)

Let f = Function y → • In f 5 6

If (Function y → Function z → • 0) True False Then 0 0
Else 0

4 $\cong$ If True Then 4 Else 8

expressions	Fb	FbS	FbX	
$4 \cong 3 + 1$	Y	Y	Y	
Let junk = $\overset{\perp}{e_1}$ In e_2 $\cong$ e_2	N	N	N	junk $\notin e_2$
$e_1; e_1; e_2 \cong e_1; e_2$	Y	N	Y	