

EFb

e, v grammars exactly from Fb

1. Infer type and equations $\Gamma \vdash e : \tau \setminus E$
2. Deductive closure
3. Check consistency
4. Substitution of types

$$'a \setminus \{ 'a = 'b \rightarrow 'c, 'c = \text{Int}, 'b \rightarrow \text{Int} = \text{Int} \rightarrow 'c \}$$

1. Inference

$$\Gamma \vdash e : \tau \setminus E$$

$$\tau ::= \text{Int} \mid \text{Bool} \mid \tau \rightarrow \tau \mid \alpha$$

$$\alpha ::= 'a \mid 'b \mid \dots$$

Variable $\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau \setminus \emptyset}$

$$E = \{ \text{Int} = 'a, \dots \}$$

Application $\frac{\Gamma \vdash e_1 : \tau \setminus E_1 \quad \Gamma \vdash e_2 : \tau' \setminus E_2 \quad \alpha \text{ fresh}}{\Gamma \vdash e_1 e_2 : \alpha \setminus E_1 \cup E_2 \cup \{ \tau = \tau' \rightarrow \alpha \}}$

Let $\frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2 \setminus E_2}{\Gamma \vdash \text{Let } x = e_1 \text{ In } e_2 : \tau_2 \setminus E_1 \cup E_2}$

↑ syntax

If $\frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \quad \Gamma \vdash e_2 : \tau_2 \setminus E_2 \quad \Gamma \vdash e_3 : \tau_3 \setminus E_3}{\Gamma \vdash \text{If } e_1 \text{ Then } e_2 \text{ Else } e_3 : \tau_2 \setminus E_1 \cup E_2 \cup E_3 \cup \left\{ \begin{array}{l} \tau_1 = \text{Bool}, \\ \tau_2 = \tau_3 \end{array} \right\}}$

$e_2 = \dots$

$$\Gamma \vdash e_2 : \text{Int} \setminus \emptyset \quad \Gamma \vdash e_3 : 'a \setminus \{ 'a = \text{Int} \}$$

$$\Gamma \vdash \text{If } e_1 \text{ Then } e_2 \text{ Else } e_3 : \alpha \setminus E_1 \cup E_2 \cup E_3 \cup \left\{ \begin{array}{l} \tau_1 = \text{Bool}, \\ \tau_2 = \tau_3 \\ \alpha = \tau_2 \end{array} \right\}$$

Step 2 by example (1st: Step 1)

$\Gamma_{ab} = \{b:'a, x:'b\}$

Function $b \rightarrow$ Function $x \rightarrow$ If b Then x Else $x+x$

$Bool \rightarrow Int \rightarrow Int$

$$\frac{\text{Var } \Gamma_{ab} \vdash b : 'a \setminus \emptyset \quad \text{Var } \Gamma_{ab} \vdash x : 'b \setminus \emptyset \quad \text{Plus } \Gamma_{ab} \vdash x+x : Int \setminus \{ 'b=Int \}}{\text{Fn } \{b:'a, x:'b\} \vdash \text{Function } x \rightarrow \text{If } b \text{ Then } x \text{ Else } x+x : 'b \rightarrow 'b \setminus \{ 'b=Int, 'a=Bool \}} \quad \alpha \text{ fresh}$$

$$\frac{\text{Fn } \{b:'a, x:'b\} \vdash \text{Function } x \rightarrow \text{If } b \text{ Then } x \text{ Else } x+x : 'b \rightarrow 'b \setminus \{ 'b=Int, 'a=Bool \}}{\emptyset \vdash \text{Function } b \rightarrow \text{Function } x \rightarrow \text{If } b \text{ Then } x \text{ Else } x+x : 'a \rightarrow 'b \rightarrow 'b \setminus \{ 'b=Int, 'a=Bool \}}$$

$'a \rightarrow 'b \rightarrow 'b \setminus \{ 'b=Int, 'a=Bool \}$

If we call w/ True:

$'c \setminus \{ 'b=Int, 'a=Bool, 'a \rightarrow 'b \rightarrow 'b = Bool \rightarrow 'c \}$

- $\tau = \tau'$ then $\tau' = \tau$
- $\tau = \tau'$ and $\tau' = \tau''$ then $\tau = \tau''$
- $\tau_1 \rightarrow \tau_2 = \tau'_1 \rightarrow \tau'_2$ then $\tau_1 = \tau'_1$ and $\tau_2 = \tau'_2$

Step 2

$$'c \setminus \left\{ \begin{array}{l} 'b=Int \\ 'a=Bool \\ 'a \rightarrow 'b \rightarrow 'b = Bool \rightarrow 'c \end{array} \right\}$$

$'b \rightarrow 'b = 'c$
 $'c = 'a \rightarrow 'a$
 $'b \rightarrow 'b = 'a \rightarrow 'a$
 $'b = 'a$

$$\left. \begin{array}{l} E \ni Int = Bool \\ E \ni Int = \tau \rightarrow \tau \\ E \ni Bool = \tau \rightarrow \tau \end{array} \right\} \leftarrow \text{Step 3 : Check}$$

$'c = 'b \rightarrow 'b = Int \rightarrow Int$

$'a \setminus \{ 'a = 'b \rightarrow 'a \}$

$$'a = 'b \rightarrow 'a = 'b \rightarrow 'b \rightarrow 'a = 'b \rightarrow 'b \rightarrow 'b \rightarrow 'a$$

$$'a = 'b$$



$$\text{Plus } \frac{\Gamma_{\tau_{e_1}: \tau_1 \setminus E_1} \quad \Gamma_{\tau_{e_2}: \tau_2 \setminus E_2}}{\Gamma_{\tau_{e_1+e_2}: \text{Int} \setminus E_1 \cup E_2 \cup \{\tau_1 = \tau_2 = \text{Int}\}}}$$

$$\text{Bad Plus } \frac{\Gamma_{\tau_{e_1}: \text{Int} \setminus E_1} \quad \Gamma_{\tau_{e_2}: \text{Int} \setminus E_2}}{\Gamma_{\tau_{e_1+e_2}: \text{Int} \setminus E_1 \cup E_2}}$$

$$\text{Int} \setminus \emptyset \quad \alpha \setminus \{\alpha = \text{Int}\}$$

$\underbrace{\quad} \quad \underbrace{\quad}$
 $1 + f \quad 2$