$\tau_1' <: \tau_1 \quad \tau_2' <: \tau_2$

$\tau_1 \to \tau_2' <: \tau_1' \to \tau_2$

If $(\tau_1' \to \tau_2') \to (\tau_3' \to \tau_4') <: (\tau_1 \to \tau_2) \to (\tau_3 \to \tau_4)$

$+ \quad - \quad - \quad +$

then

$(\tau_3' \to \tau_4') <: (\tau_3 \to \tau_4)$ $\quad - \longrightarrow +$

$(\tau_1 \to \tau_2) <: (\tau_1' \to \tau_2')$

$\tau_4' <: \tau_4$

$\tau_3 <: \tau_3'$

$\tau_2 <: \tau_2'$

$\tau_1' <: \tau_1$

Type Inference

Type inference: determine types of e.g. expressions based upon structure, content, etc. w/o being explicit annotation

Pros: succinct

Cons: confusing

inf $x = 4 + 5 + 6;$

EFb — syntax = Fb

equational

$\{ x = y,\ y = z \}$
$x = z$

1. Infer types and equations
2. Deductive closure of equations
3. Check equations
4. Substitution

Step 1: Inference

$\tau ::= \text{Int} \mid \text{Bool} \mid \tau \to \tau \mid \alpha$ types

$\alpha ::= \text{'a} \mid \text{'b} \mid \cdots$ type variables

$E ::= \{ \tau = \tau, \ldots \}$

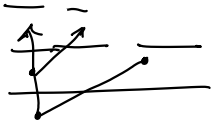
$\Gamma \vdash e : \tau \setminus E$

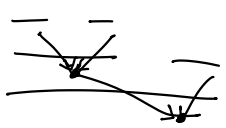
Assuming Γ, e has type τ where the equations E hold.

$\emptyset \vdash e : \text{Int} \setminus \emptyset$

$\emptyset \vdash e : \underbrace{\alpha \setminus \{\alpha = \text{Int}\}}_{\text{equational type}}$

$\frac{Fb}{S}$ 

$\frac{TFb}{\Gamma}$ 

$\frac{EFb}{E}$ 

$$\text{Int}\ \frac{v \in \mathbb{Z}}{\Gamma \vdash v : \text{Int} \setminus \emptyset}$$

$$\text{Plus}\ \frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \qquad \Gamma \vdash e_2 : \tau_2 \setminus E_2}{\Gamma \vdash e_1 + e_2 : \text{Int} \setminus E_1 \cup E_2 \cup \{\tau_1 = \text{Int}, \tau_2 = \text{Int}\}}$$

$$\text{Equal}\ \frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \qquad \Gamma \vdash e_2 : \tau_2 \setminus E_2}{\Gamma \vdash e_1 = e_2 : \text{Bool} \setminus E_1 \cup E_2 \cup \{\tau_1 = \text{Int}, \tau_2 = \text{Int}\}}$$

$$\text{Plus}\ \frac{\text{Int}\ \dfrac{1 \in \mathbb{Z}}{\emptyset \vdash 1 : \text{Int} \setminus \emptyset} \qquad \text{Int}\ \dfrac{2 \in \mathbb{Z}}{\emptyset \vdash 2 : \text{Int} \setminus \emptyset}}{\emptyset \vdash 1 + 2 : \text{Int} \setminus \{\text{Int} = \text{Int}\}}$$

$\{4, 4\} = \{4\}$

$$\text{Function}\ \frac{\Gamma, x : \alpha \vdash e : \tau \setminus E \qquad \alpha \text{ is fresh}}{\Gamma \vdash (\text{Function } x \to e) : \alpha \to \tau \setminus E}$$

$$\frac{\dfrac{}{\{x : 'a\} \vdash x : 'a \setminus \emptyset} \qquad \dfrac{}{\{x : 'a\} \vdash 1 : \text{Int} \setminus \emptyset}}{\dfrac{\{x : 'a\} \vdash x + 1 : \text{Int} \setminus \{'a = \text{Int}, \text{Int} = \text{Int}\}}{\emptyset \vdash \text{Function } x \to x + 1 : 'a \to \text{Int} \setminus \{'a = \text{Int}, \text{Int} = \text{Int}\}}}$$

$$\frac{\dfrac{}{\emptyset \vdash 1 : \text{Int} \setminus \emptyset} \qquad \dfrac{}{\emptyset \vdash \text{True} : \text{Bool} \setminus \emptyset}}{\emptyset \vdash 1 + \text{True} : \text{Int} \setminus \{\text{Int} = \text{Int}, \text{Bool} = \text{Int}\}}$$

Step 2

1. If $\{\tau_1 = \tau_2, \tau_2 = \tau_3\} \subseteq E$ then add $\tau_1 = \tau_3$.
2. If $\{\tau_1 = \tau_2\} \subseteq E$ then add $\tau_2 = \tau_1$.
3. If $\{\tau_1 \to \tau_2 = \tau_1' \to \tau_2'\} \subseteq E$ then add $\tau_1 = \tau_1'$ and $\tau_2 = \tau_2'$.

Step 3

E is <u>bad</u> if $\text{Int} = \text{Bool}$, $\text{Int} = \tau \to \tau'$, $\text{Bool} = \tau \to \tau'$