$$\Gamma \vdash e : \tau$$

FbR

$$(\text{Function } x \to x.a) \quad \{a = 5\} \Longrightarrow 5$$
$$(\text{Function } x \to x.a) \quad \{a = 5; b = 8\} \Longrightarrow 5$$

TFbR

$$(\text{Function } x : \{a : \text{Int}\} \to x.a) \quad \{a = 5\} \Longrightarrow 5 \quad : \text{Int}$$
$$(\text{Function } x : \{a : \text{Int}\} \to x.a) \quad \{a = 5; b = 8\} \Longrightarrow 5 \quad : \text{"}$$

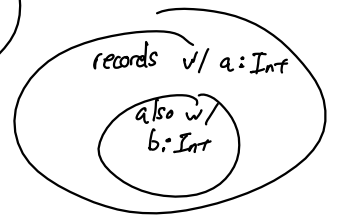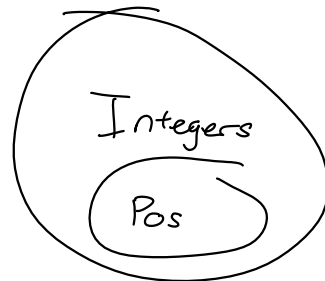$$\underbrace{\{a : \text{Int}\} \to \text{Int}} \qquad \underbrace{\{a : \text{Int}, b : \text{Int}\}}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 \, e_2 : \tau_2}$$

Subtyping

Type — set of values
Subtype — subset of values

Type — set of promises
Subtype — more promises

use $\{a : \text{Int}, b : \text{Int}\}$
expect $\{a : \text{Int}\}$

$\{a : \text{Int}, b : \text{Int}\}$ is a subtype of $\{a : \text{Int}\}$

Subtypes

$$``<:"\ \approx\ ``\subseteq"\ \approx\ ``\leq"$$

$$e \Rightarrow v$$
$$\Gamma \vdash e : \tau$$
$$\tau <: \tau$$

$$\tau <: \tau$$

$$\{a:Int, b:Int\} <: \{a:Int\} \qquad\qquad Int <: Int$$

Reflexivity $\dfrac{}{\tau <: \tau}$

$$\begin{aligned}\{l_1:\tau_1,...,l_n:\tau_n\} &= \\ \{l_n:\tau_n,...,&l_1:\tau_1\}\end{aligned}$$

Transitivity $\dfrac{\tau_1 <: \tau_2 \quad \tau_2 <: \tau_3}{\tau_1 <: \tau_3}$

$$\{a:Int\} \not<: \{b:Int\}$$

$$\{a: \{b:Int, c:Bool\}\} <: \{a: \{c:Bool\}\}$$

~~**Symmetry** $\dfrac{\tau_1 <: \tau_2}{\tau_2 <: \tau_1}$~~

$$myrec.a.c$$

Record Subtype Width
$$\dfrac{\tau_2' = \{l_1:\tau_2,...,l_n:\tau_n\} \quad \tau_2' = \{l_1:\tau_2, ..., l_n:\tau_n, ..., l_m:\tau_m\}}{\tau_2' <: \tau_1'}$$

Record Depth Subtype
$$\dfrac{\tau_1 = \{l_1:\tau_1',...,l_n:\tau_n'\} \quad \tau_2 = \{l_1:\tau_1'',...,l_n:\tau_n''\} \quad \begin{matrix}\tau_1' <: \tau_1'' \\ ... \\ \tau_n' <: \tau_n''\end{matrix}}{\tau_1 <: \tau_2}$$

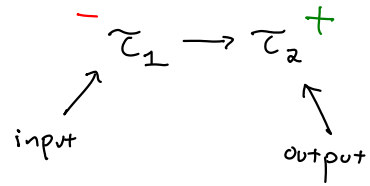$$\{a:Int, b:\{x:Int, y:Int\}\} <: \{a:Int, b:\{x:Int\}\} <: \{a:Int\}$$

Function Subtyping

$$\frac{\tau_2 <: \tau_2' \qquad \tau_1' <: \tau_1}{\tau_1 \longrightarrow \tau_2 \ <: \ \tau_1' \longrightarrow \tau_2'}$$

$$\{\} \longrightarrow \{p:Int, q:Bool\}$$
$$<: \{a:Int\} \longrightarrow \{p:Int\}$$

functions have
  Covariant return types
  contravariant parameter type

$$\overset{-}{\tau_1} \longrightarrow \overset{+}{\tau_2}$$

input   output

$$\{a:Int\} \longrightarrow \{a:Int\}$$

I will accept __ANY__ value which is a record containing an a:Int field

my output is a record containing __at least__ {a:Int}

$$(\overset{-}{\tau_1} \to \overset{+}{\tau_2}) \to \tau_3 \ <: \ (\overset{-}{\tau_1'} \to \overset{+}{\tau_2'}) \to \tau_3$$

$$\overset{-}{} \qquad \overset{+}{} \qquad \overset{-}{} \qquad \overset{+}{}$$

$$\overset{+}{} \quad \overset{-}{} \qquad \overset{+}{}$$

Subtype $$\frac{\Gamma \vdash e : \tau \qquad \tau <: \tau'}{\Gamma \vdash e : \tau'}$$

$$(\text{Function } x : \{a:Int\} \longrightarrow x.a) \quad \{a=5; b=8\}$$

$$\{a:Int\} \longrightarrow Int \qquad\qquad \{a:Int, b:Int\}$$

$$\{a:Int\}$$

STFbR