Types

A type is a set of values.

$v ::= 0 \mid 1 \mid -1 \mid \cdots$ → Int

$\mid$ True $\mid$ False $\mid$ → Bool

Function $x \to e$

$\tau ::= \text{Int} \mid \text{Bool} \mid \cdots$

$\Gamma \vdash e : \tau$

Assuming Γ, e has type τ.

$e \Rightarrow v$

Evaluating e should produce v

If e is evaluated, then either $e \Rightarrow v$ for $v \in \tau$ or e loops forever.

$$\frac{\Gamma \vdash e_1 : \text{Bool} \quad \Gamma \vdash e_2 : \text{Bool}}{\Gamma \vdash e_1 \text{ And } e_2 : \text{Bool}}$$

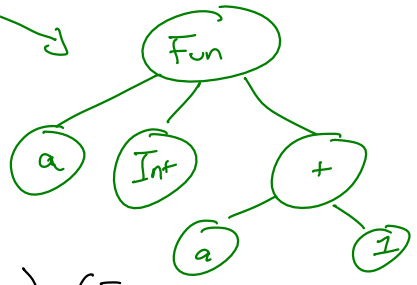
Function $n : \text{Int} \to n = 0$

$$\frac{\Gamma, x:\tau \vdash e : \tau'}{\Gamma \vdash (\text{Function } x:\tau \to e) : \tau \to \tau'}$$

Function $a : \text{Int} \to a + 1$

$v ::= \cdots \mid$ Function $x:\tau \to e$

$\tau ::= \text{Int} \mid \text{Bool} \mid \tau \to \tau$



$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\ e_2 : \tau_2}$$

$$\frac{x:\tau \in \Gamma}{\Gamma \vdash x : \tau}$$

TFb ↑ no Let Rec

TcFb ↑ yes Let Rec

Cannot type Y-combinator

(Function $x : \tau \to x\ x$) (Function $x:\tau \to x\ x$)

$\tau_\omega = \tau_\omega \to \tau_2 \rightleftharpoons \tau_\omega \to \tau_\omega \to \tau_2 = \cdots$

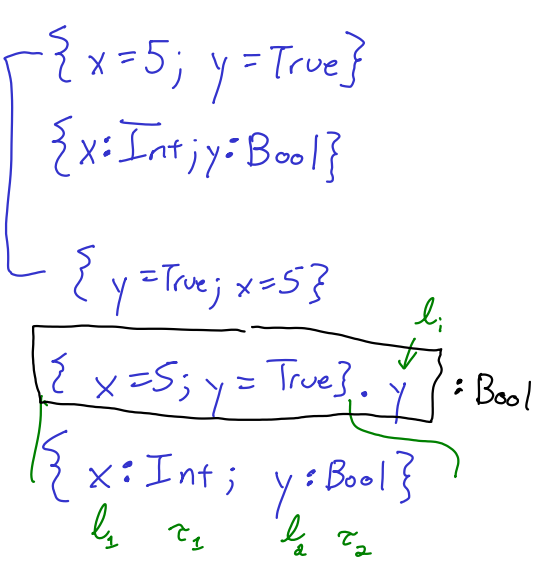
TFb is normalizing

TFbR

$$e ::= \cdots \mid \{\ell = e, \ldots\} \mid e.\ell$$
$$v ::= \cdots \mid \{\ell = v, \ldots\}$$
$$\tau ::= \cdots \mid \{\ell : \tau, \ldots\}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \cdots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash \{\ell_1 = e_1; \cdots; \ell_n = e_n\} : \{\ell_1 : \tau_1, \cdots, \ell_n : \tau_n\}}$$

$$\frac{\Gamma \vdash e : \{\ell_1 : \tau_1, \cdots, \ell_n : \tau_n\} \quad i \in \{1, \ldots, n\}}{\Gamma \vdash e.\ell_i : \tau_i}$$



TFbS

$e ::= \ldots \mid \text{Ref } e \mid e := e \mid !e$

$v ::= \ldots \mid c$

$\tau ::= \ldots \mid \text{Ref } \tau$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \text{Ref } e : \text{Ref } \tau}$$

$$\frac{\Gamma \vdash e_1 : \text{Ref } \tau_1 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 := e_2 : \tau_1}$$

$$\frac{\Gamma \vdash e : \text{Ref } \tau}{\Gamma \vdash !e : \tau}$$

$$\frac{\langle S_1, e_1 \rangle \Rightarrow \langle S_2, c \rangle \quad \langle S_2, e_2 \rangle \Rightarrow \langle S_3, v \rangle}{\langle S_1, e_1 := e_2 \rangle \Rightarrow \langle S_3\{c \mapsto v\}, \boxed{?} \rangle}$$

v

FbS

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Let r = Ref 0 In
r := True;
r := 4;
!r + 2
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Ref 4 := 5