$v ::= \%\%_n \mid \text{True} \mid \text{False} \mid \text{Function } x \to e$

$$e \Rightarrow v$$

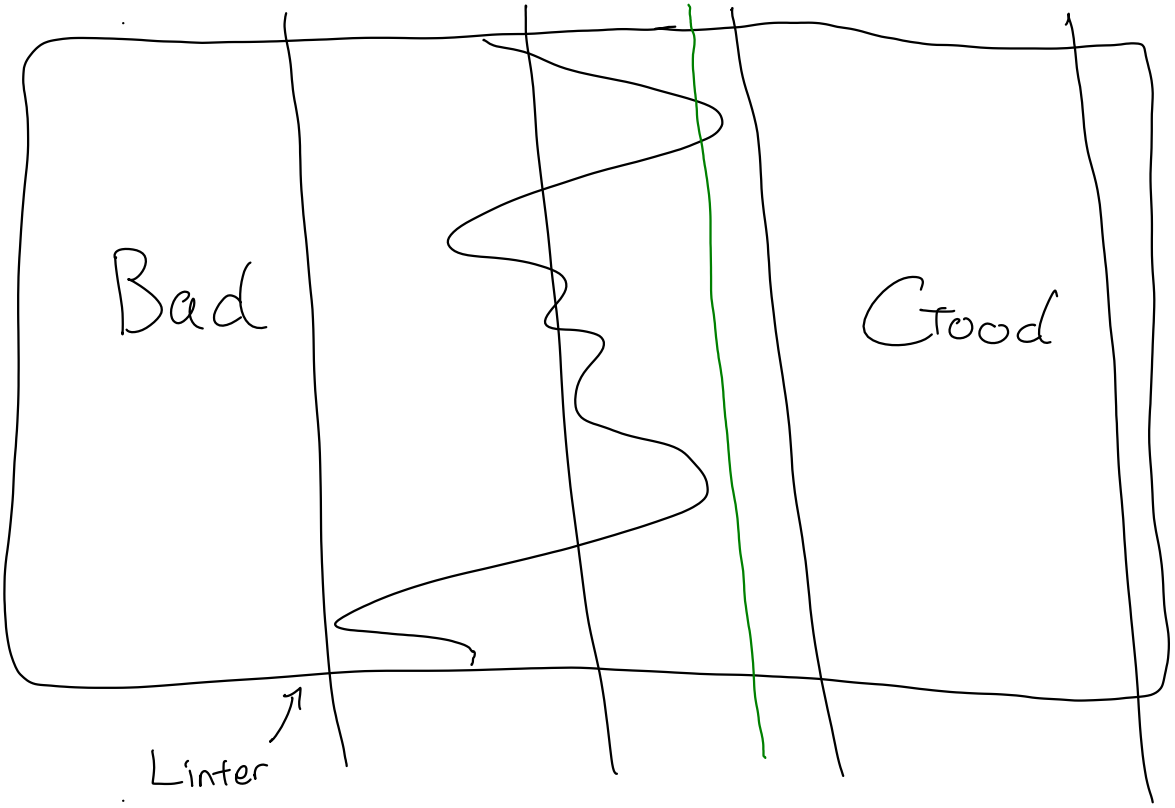
$$\langle S, e \rangle \Rightarrow \langle S, v \rangle$$

Type System

A type system is a tool which analyzes (which) part of a program to prove things (what) about (if) before (when) it runs.

When?
- Compile-time → C, C++, Java, OCaml, etc.
- Runtime → Python
- Not → C array checking

prove types of values in variables?

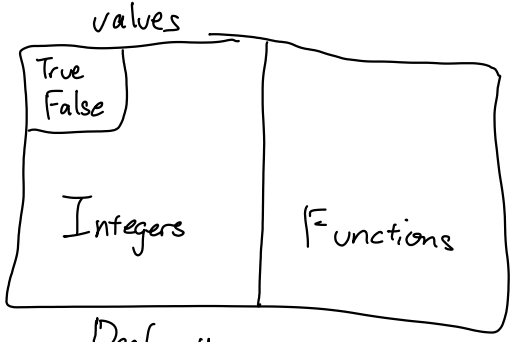


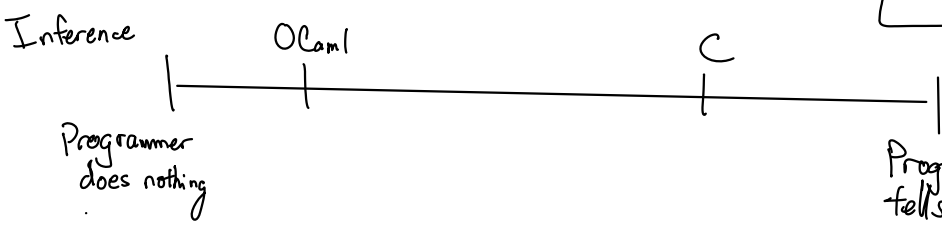

```
int x = (int) y;
```

TFb

typed Fb

Type is a set of values





int x = 1+f();

TFb: every variable declaration has a type

$e ::= v \mid \cdots \mid \text{Let } x:\tau = e_1 \text{ In } e_2$

$v ::= \cdots \mid \text{Function } x:\tau \to e$

$\tau ::= \text{Int} \mid \text{Bool} \mid \tau \to \tau$

$\text{Int} \to (\text{Int} \to \text{Bool})$

Let f : Int → Int =
Function x : Int →
x+1

In
...

TFb Let $\dfrac{e_1 \Rightarrow v_1 \quad e_2[v_1/x] \Rightarrow v_2}{\text{Let } x:\tau = e_1 \text{ In } e_2 \Rightarrow v_2}$

$e \Rightarrow v$

$\Gamma \vdash e : \tau$

Γ : set of assumptions

Assuming Γ, then either $e \Rightarrow v$ where $v \in \tau$ or e loops forever.
(e does not get stuck)

$\emptyset \vdash (\text{Let } x = 3 \text{ In } 1 + x) : \text{Int}$

$\Gamma ::= \{ x:\tau, \ldots \}$

✓ $\{x : \text{Int}\} \vdash x+1 : \text{Int}$

✗ $\emptyset \vdash x+1 : \text{Int}$

$\emptyset \vdash \text{Let } x : \text{Int} = 3 \text{ In } x + 1 : \text{Int}$ $\quad\quad$ $\Gamma \vdash e : \tau$

$$\frac{v \in \mathbb{V}}{\Gamma \vdash v : \text{Int}} \qquad \frac{}{\Gamma \vdash \text{True} : \text{Bool}} \qquad \frac{}{\Gamma \vdash \text{False} : \text{Bool}}$$

Point use $\Rightarrow$

$$\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1, v_2 \in \mathbb{V}}{\Gamma \vdash e_1 + e_2 : \text{Int}} \qquad \frac{\Gamma \vdash e_1 : \text{Int} \quad \Gamma \vdash e_2 : \text{Int}}{\Gamma \vdash e_1 + e_2 : \text{Int}}$$

$S\{c \mapsto v\}$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \text{Let } x : \tau_1 = e_1 \text{ In } e_2 : \tau_2} \qquad \frac{(x : \tau) \in \Gamma}{\Gamma \vdash x : \tau}$$

$$\text{Function}\ \frac{\Gamma, x:\tau \vdash e : \tau'}{\Gamma \vdash \text{Function}\ x:\tau \to e : \tau \to \tau'}$$

Function x: Bool → x+1

Application