

Let $y_c = \text{Function } f \rightarrow \dots \text{ In}$ Function recurse \rightarrow Function $x \rightarrow \dots$

Let $rsum' = \text{Function recurse} \rightarrow \text{Function } n \rightarrow$
If $n = 0$ Then 0 Else
 $n + \text{recurse}(n-1)$
In $\text{recurse}(n-1) + n$

Let $rsum = y_c \ rsum' \text{ In}$

Tuples

(4, 8)

Encoding

destructor {
 fst
 Snd
 constructor { pair

Encoding Let

Let $x = e_1$ In e_2
 $(\text{Function } x \rightarrow e_2) e_1$

$$\begin{array}{l} \text{pair } 4 \ 8 \stackrel{\text{def}}{=} (\text{Function } x \rightarrow \text{Function } y \rightarrow ?) \\ \text{fst } ? \stackrel{\text{def}}{=} (\text{Function } p \rightarrow ?) ? \end{array}$$

Church 2 $\equiv (\text{Function } f \rightarrow \text{Function } x \rightarrow f(x))$

$$\text{pair } 4 \ 8 \quad (4,8)$$



Encode pairs as functions w/ values that they produce when asked

$$(4,8) \quad \text{Function } z \rightarrow \text{If } z \text{ Then } 4 \text{ Else } 8$$

Let pair = Function $x \rightarrow$ Function $y \rightarrow$ (Function $z \rightarrow$ If z Then x Else y)

In
 Let fst = Function $p \rightarrow p$ True In

Advantage: no need to change Fb

Sometimes: more power

Disadvantage: more power

less immediately descriptive

Not encoding pairs:

Define pairs

Fb P.

(1+3, True)
First (1+3)

$e ::= \dots | (e, e) \mid \text{First } e \mid \text{Second } e$

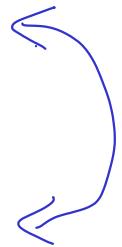
$v ::= \dots | (v, v)$

$$\text{Pair Rule} \quad \frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2}{(e_1, e_2) \Rightarrow (v_1, v_2)}$$

Not
First
Rule

$$\frac{e_1 \Rightarrow v_1}{\text{First}(e_1, e_2) \Rightarrow v_1}$$

$$\text{First Rule} \quad \frac{e \Rightarrow (v_1, v_2)}{\text{First } e \Rightarrow v_1}$$



First (3, 4 5) \Rightarrow

If True Then 3 Else 4 5 \Rightarrow 3

Similar to encoding

Lists (encoding)

$$[1, 4, 8] \stackrel{\text{def}}{=} (3, (1, (4, 8)))$$

↓
 $\stackrel{\text{def}}{=} (1, \text{True}, (4, \text{True}, ($
(element, empty, rest)
 $8, \text{True}, (0, \text{False}, 0)))))$
 $(1, \text{True})$, $((0, \text{False})$

Let $\text{empty} = ((\text{o}, \text{False}), \text{o})$ In

Let $\text{cons} = \text{Function head} \rightarrow \text{Function tail} \rightarrow ((\text{head}, \text{True}), \text{tail})$ In

Let $\text{hd} = \text{Function } (\text{st} \rightarrow \text{First}(\text{First st}))$ In

Let isempty = Function $\text{lst} \rightarrow \text{Not } (\text{Second } (\text{First } \text{lst}))$