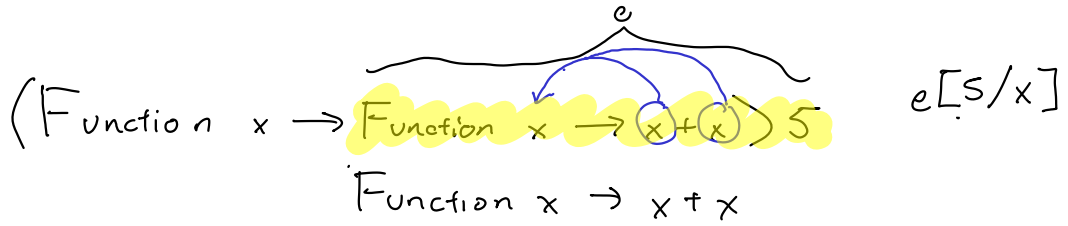
# Variable Substitution

$e[v/x]$ in $e$, replace every* $x$ with $v$

$e$ ← might be free in $x$

(Function x → (Function y → x + y)) 5 ⟶ $e[5/x]$

Function y → 5 + y ⟵ $e[5/x]$




# If-Then-Else

If True Then 4 Else False + 1

$e ::= \ldots \mid$ If $e$ Then $e$ Else $e \mid \cdots$

If-True

$$\frac{e_1 \Rightarrow \text{True} \quad e_2 \Rightarrow v_2}{(\text{If } e_1 \text{ Then } e_2 \text{ Else } e_3) \Rightarrow v_2}$$

BAD IF

$$\frac{e_1 \Rightarrow \text{True} \quad e_2 \Rightarrow v_2 \quad e_3 \Rightarrow v_3}{\text{If } e_1 \text{ Then } e_2 \text{ Else } e_3 \Rightarrow v_2}$$

# Application (Calling Functions)

$$e ::= \ldots \mid e\ e \mid \ldots$$

1. Remove body from fn
2. Perform substitution $e[v/x]$
3. Evaluate result

Application Rule

$$\frac{e_1 \Rightarrow \text{Function } x \to e \quad e_2 \Rightarrow v_1 \quad e[v_1/x] = e' \quad e' \Rightarrow v_2}{e_1\ e_2 \Rightarrow v_2}$$

$e[v/x]$ Subst $(e, x, v)$

$$e ::= \ldots \mid \text{Let } x = e_1 \text{ In } e_2 \mid \ldots$$

$$\frac{e_1 \Rightarrow v_1 \quad e_2[v_1/x] \Rightarrow v_2}{\text{Let } x = e_1 \text{ In } e_2 \Rightarrow v_2}$$

"encoding"

$$\text{Let } x = e_1 \text{ In } e_2 \equiv (\text{Function } x \to e_2)\ (e_1)$$

# Let Rec

$$\frac{e_2\,[\text{Function } x \hookrightarrow e_1[\text{Function } x \to \text{Let Rec } x'\ x = e_1 \text{ In } x'\ x/x']/x'] \Rightarrow v}{\text{Let Rec } x'\ x = e_1 \text{ In } e_2 \Rightarrow v}$$

$x'$ $x$ — $e_1$

Let Rec f y = If y = 0 Then 0 Else y + f(y - 1)
In
f 5 — $e_2$

$e_1$: If y = 0 Then 0 Else y + (Function y → Let Rec f y = $e_1$ In f y) (y - 1)

# Self-Passing Recursion

```
Let sum = Function n ->
            If n = 0 Then 0 Else
              n + sum (n-1)

In
 sum 5
```

```
If 5=0 Then 0 Else
     5 + sum (5-1)
```

```
5 + sum (5-1)
```

Function

```
Let sum' = Function self -> (Function n ->
             If n=0 Then 0 Else
               n + self self (n-1) )
```

self=sum'

```
In
 sum' sum' 5

Let sum = sum' sum' In
 sum 5
```

(2+3)+4

# Properties of Fb

Deterministic? yes $\quad \forall e.\, \exists v.\ e \Rightarrow v$

Normalizing? no $\quad \{1\} \in P$

Russel's Paradox: Let P be the set of all sets that do not contain themselves.

$$P \stackrel{?}{\in} P$$