Relations

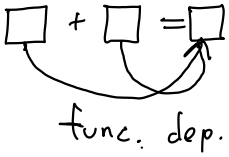
~~$\square \sim \square * \square$~~

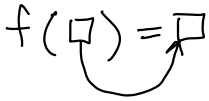
$2 \sim 4 * 6$ } ✓ $\quad 0 \sim 7 * 3$ ✗

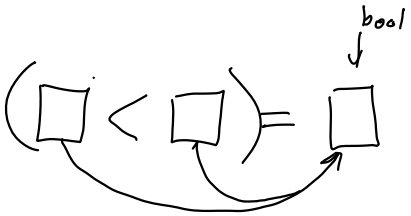
$1 \sim 2 * 3$ }

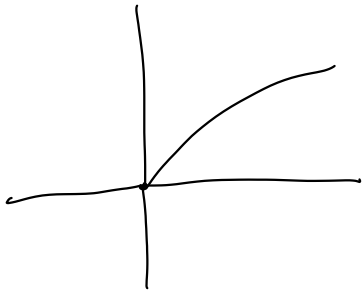
Relation

Function





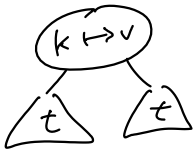


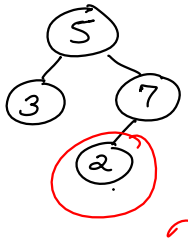


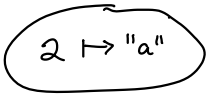
Rules of Inference

$$\frac{}{n \leq n} \qquad \frac{n_1 \leq n_2}{dec(n_1) \leq n_2} \qquad \frac{n_1 \leq n_2}{n_1 \leq inc(n_2)}$$

maps $\square$ to $\square$

$t ::= \in \mid$ 



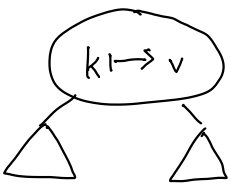


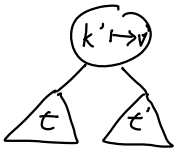
maps 2 to "a"

$\forall k'' \mapsto v'' \in t'.\ k'' > k'$

FOUND $\dfrac{}{k \mapsto v}$ maps k to v

LEFT $\dfrac{k < k' \quad t}{k' \mapsto v}$ maps k to v





$e \Longrightarrow v$

Operational Semantics

True
False
True And True
True And False
(True Or (True Or False)) Or True

} ex. of boolean expressions

EBNF

v ::= True | False
e ::= v | e And e | e Or e

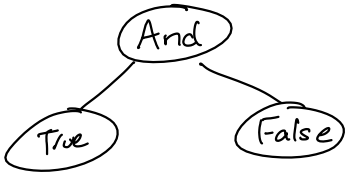
Concrete Syntax

Abstract Syntax Trees

True

False

True And False



True And False $\Longrightarrow$ v

(True And False) Or (False Or True)

These don't scale

$$\frac{}{e \text{ And False} \Longrightarrow \text{False}} \qquad \frac{}{\text{True Or } e \Longrightarrow \text{True}} \qquad \frac{}{\text{False} \Longrightarrow \text{False}}$$

$$\frac{}{\text{False And } e \Longrightarrow \text{False}} \qquad \frac{}{e \text{ Or True} \Longrightarrow \text{True}}$$

$$\frac{}{\text{True And True} \Longrightarrow \text{True}} \qquad \frac{}{\text{False Or False} \Longrightarrow \text{False}}$$

$e \Rightarrow \boxed{v}$

$v ::= \text{True} \mid \text{False}$

$e ::= v \mid e \text{ And } e \mid e \text{ Or } e$

$$\frac{}{v \Rightarrow v} \qquad \frac{e_1 \Rightarrow \text{True} \quad e_2 \Rightarrow \text{True}}{e_1 \text{ And } e_2 \Rightarrow \text{True}} \qquad \frac{e_1 \Rightarrow \text{False}}{e_1 \text{ And } e_2 \Rightarrow \text{False}}$$

. . . .

Deterministic: $\forall e.$ if $e \Rightarrow v_1$ and $e \Rightarrow v_2$ then $v_1 = v_2$

Normalizing: $\forall e. \exists v. e \Rightarrow v$

Converge: given e, $\exists v. e \Rightarrow v$

Diverge: not converge