Intro to Logic.

proposition not "function call"

$\forall$ for all . $\forall x \in X.\ P(x)$

$\exists$ exists .

Discuss truth

$\forall x \in X\ .\ P(x)$

&& ||
$\wedge$ $\vee$
and or

Statements

It is cloudy.
I am flying.

$P(d) =$ if cloudy(d) then raining(d)

$\forall$days. If cloudy(day) then raining(day).
$\forall$days. P(day)

If I have no work and
 if is a weekend
then I sleep in.

$\text{work} = \emptyset \wedge \text{weekend(today)} \Longrightarrow \text{sleepIn}$

Quantifier Ordering

$\exists x.\ \forall y.\ x > y.$ $\leftarrow$

$\forall y.\ \exists x.\ x > y.$

Inference Rules and Proof Systems

ϟ △ □

$$1\ \dfrac{ϟ}{ϟ\square} \qquad 2\ \dfrac{\square}{\square\square} \qquad 3\ \dfrac{ϟ\square}{ϟ\triangle} \qquad 4\ \dfrac{ϟ\square}{ϟ\triangle\square} \qquad 5\ \dfrac{ϟ\triangle}{\triangle\triangle} \qquad 6\ \dfrac{}{ϟ}$$

prove △△

ϟ then ϟ□ then ϟ△ then △△

$$\begin{array}{ll} 6 & \overline{\quad} \\ & ϟ \\ 1 & \overline{\quad\quad} \\ & ϟ\square \\ 3 & \overline{\quad\quad} \\ & ϟ\triangle \\ 5 & \overline{\quad\quad} \\ & \triangle\triangle \end{array}$$

△△ because
ϟ△ because
ϟ□ because
ϟ axiomatically

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metavariable

$S \stackrel{\text{def}}{=}$ any sequence of ϟ, △, or □

$$1\ \dfrac{}{ϟ} \qquad 2\ \dfrac{ϟ}{ϟ\square\square} \qquad 3\ \dfrac{S\square}{S\triangle} \qquad 4\ \dfrac{S\square}{S\triangle\triangle} \qquad 5\ \dfrac{ϟS}{\square S} \qquad 6\ \dfrac{\square S\triangle}{S}$$

prove “ ”

$$\begin{array}{c} \overline{\quad} \\ ϟ \\ \hline ϟ\square\square \\ \hline \square\square\square \\ \hline \square\square\triangle\triangle \\ \hline \square\triangle \\ \hline \epsilon \end{array}$$

prove: □

$$\begin{array}{c} ϟ \\ \hline ϟ\square\square \\ \hline \square\square\square \\ \hline \square\square\triangle \\ \hline \square \end{array}$$

$$\begin{array}{c} \overline{\quad} \\ ϟ \\ \hline \square \end{array}$$

1 $\dfrac{}{ϟ}$ 2 $\dfrac{S}{S\square\square}$ 3 $\dfrac{S_1 ϟ S_2}{S_1 \triangle ϟ S_2}$ 4 $\dfrac{S_1 \square\triangle S_2}{S_1 S_2}$ 5 $\dfrac{S_1 \qquad S_2}{S_1 ϟ S_2}$

$\triangle\triangle ϟϟϟ \square\square$

$$5\;\dfrac{3\;\dfrac{3\;\dfrac{1\;\dfrac{}{ϟ}}{\triangle ϟ}}{\triangle\triangle ϟ} \qquad 2\;\dfrac{1\;\dfrac{}{ϟ}}{ϟ\square\square}}{\triangle\triangle ϟϟϟ\square\square}$$

$\leq$

$f(x)$

function : mapping from each S onto one T
relation : mapping from each S onto any subset of T

$2 \leq 4$

$$\dfrac{}{n \leq n} \qquad \dfrac{n_1 \leq n_2}{dec(n_1) \leq n_2} \qquad \dfrac{n_1 \leq n_2}{n_1 \leq inc(n_2)}$$

metavariable t is either ϵ or (k, v, t_1, t_2)

$lookup(t, k) = v.$

$$\dfrac{}{lookup((k, v, t_1, t_2), k) = v}$$

$$\dfrac{lookup((k, v, t_1, t_2), k) > v}{lookup(t_1, k) = v}$$

$$\dfrac{lookup(t_1, k) = v \quad k < k_0}{lookup((k_0, v_0, t_1, t_2), k) = v}$$