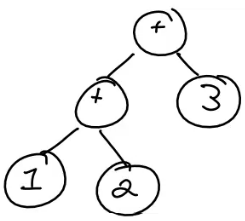
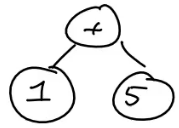
Operational Equivalence

$1+2+3$ $\qquad\qquad$ $1+5$





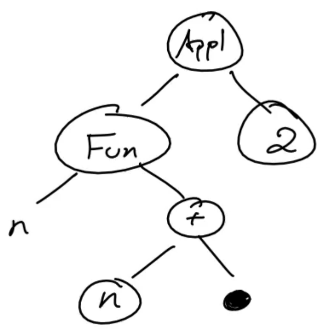
Proving operational equivalence is difficult

"Context" is an expression with exactly one subexpression replaced by a "hole" $\bullet$

C

(Function $n \to n + \bullet$) 2

$$1+2+3 \overset{\text{cong}}{\cong} 1+5$$



e is <u>operationally equivalent</u> to e'
iff $\forall C.\ C[e] \Rightarrow v$ iff $C[e'] \Rightarrow v'$

(Function $n \to n + (1+2+3)$) $2 \Rightarrow 8$

(Function $n \to n + (1+5)$) $2 \Rightarrow 8$

Context examples: goal is to evaluate to False

(Function $n \to \bullet$) True $\qquad$ (Not n)

$\bullet$ $\bigcirc$ $\qquad$ (Function $x \to x = 2$)

(Function $f \to$ Not $(f\ 4\ 2)$) $\bullet$ $\qquad$ (Function $x \to$ Function $y \to x = y + 2$)

1. Any expression
2. Only expressions not containing True or False

$e \cong e'$ iff

$\forall C.\ C[e] \Rightarrow v$ iff $C[e'] \Rightarrow v'$

$4 \not\cong 5$

$x \not\cong x + 1 - 1$

$(\text{Function } x \to z) \not\cong (\text{Function } x \to y)$

For each pair, find a C that behaves differently..

$(\text{Function } x \to \text{If } x = 4 \text{ Then } 4 \text{ Else } x\ 0)\ \bullet$

$C[4] \Rightarrow 4$

$C[5] \not\Rightarrow$

$\text{Let } x = \text{True In } \bullet$

$C[x] \Rightarrow \text{True}$

$C[x+1-1] \not\Rightarrow$

$(\text{Function } z \to \bullet)\ 1\ \ 2$

$C[\text{Function } x \to z] \Rightarrow 1$

expressions	Fb	FbS	FbX
$e_1 + e_2$?$\cong$ $e_2 + e_1$	Y	N	N
$\text{Let junk} = e_1 \text{ In } e_2$?$\cong$ e_2	N	N	N
$e_1 ; e_2$?$\cong$ $e_1 ; e_1 ; e_2$	Y	N	Y