EFb Type Inferencer

$e ::= \overbrace{\ldots}^{Fb}$

$v ::= \underbrace{\ldots}_{Fb}$

$\tau ::= Int \mid Bool \mid \tau \to \tau \mid \alpha$

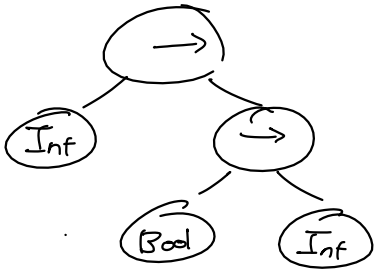
$q ::= \tau = \tau$

$E ::= \{q, \ldots\}$

$\{Int = 'a,\ Bool = 'b,\ 'a = 'q,\ 'q = 'b\}$

1. Type derivation: $\Gamma \vdash e : \tau \backslash E$
2. Deductive closure
3. Check inconsistencies
4. Type substitution

$Int \to Bool \to Int$



2. Deductive Closure

1. If $\tau_1 = \tau_2$ then $\tau_2 = \tau_1$.
2. If $\tau_1 = \tau_2$ and $\tau_2 = \tau_3$ then $\tau_1 = \tau_3$.
3. If $\tau_1 \to \tau_2 = \tau_3 \to \tau_4$ then $\tau_1 = \tau_3$ and $\tau_2 = \tau_4$.

3. Consistency Checking

1. If $Int = Bool \in E$, !!
2. If $Int = \tau \to \tau' \in E$, !!
3. If $Bool = \tau \to \tau' \in E$, !!
4. Otherwise, :)

$'a \backslash \{'a = 'b \to 'b,\ 'b = Int,\ Int = 'b,\ 'b \to 'b = 'a,$
$'a = 'a,\ 'b = 'b,\ Int = Int,\ 'b \to 'b = 'b \to 'b\}$

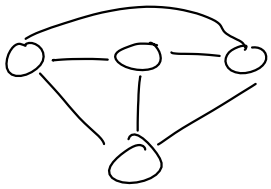
$'b \to 'b$

$Int \to Int$ Function $a \to 5$

$'_w \to Int$

4. Type Substitution

1. If type is a fn, recurse on both subtrees
2. If type is concrete and primitive (Int or Bool), return it
3. If type is a var and an equation between it and a concrete type (Int, $Bool$, $\to$), recurse on and return concrete type.
4. If type is a var and no concrete equations exist, use the lexicographically least variable



Infer a type for (Function n → n+1) 4.

$$\text{Int}\ \frac{}{\Gamma \vdash \mathbb{Z} : \text{Int} \setminus \emptyset} \qquad \text{Plus}\ \frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \qquad \Gamma \vdash e_2 : \tau_2 \setminus E_2}{\Gamma \vdash e_1 + e_2 : \text{Int} \setminus E_1 \cup E_2 \cup \{\text{Int} = \tau_1, \text{Int} = \tau_2\}}$$

$$\text{Var}\ \frac{(x:\tau) \in \Gamma}{\Gamma \vdash x : \tau \setminus \emptyset} \qquad \text{Function}\ \frac{\Gamma, x : \alpha \vdash e : \tau \setminus E \quad \alpha \text{ fresh}}{\Gamma \vdash \text{Function } x \to e : \alpha \to \tau \setminus E}$$

$$\text{Application}\ \frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \quad \Gamma \vdash e_2 : \tau_2 \setminus E_2 \quad \alpha \text{ fresh}}{\Gamma \vdash e_1\ e_2 : \alpha \setminus E_1 \cup E_2 \cup \{\tau_1 = \tau_2 \to \alpha\}}$$

(1)

$$\dfrac{\dfrac{\dfrac{\{n : 'p\} \vdash n : 'p \setminus \emptyset \qquad \{n : 'p\} \vdash 1 : \text{Int} \setminus \emptyset}{\{n : 'p\} \vdash n+1 : \text{Int} \setminus \{\text{Int} = 'p,\ \text{Int} = \text{Int}\}}}{\emptyset \vdash (\text{Function } n \to n+1) : 'p \to \text{Int} \setminus \{\text{Int} = 'p,\ \text{Int} = \text{Int}\}} \qquad \emptyset \vdash 4 : \text{Int} \setminus \emptyset}{\emptyset \vdash (\text{Function } n \to n+1)\ 4 : 'w \setminus \{\text{Int} = 'p,\ \text{Int} = \text{Int},\ 'p \to \text{Int} = \text{Int} \to 'w\}}$$

4 : True

$'p \to \text{Int} = \text{Int} \to 'w$: Bool

(2)

$\text{Int} = 'p$

$\text{Int} = \text{Int}$

$'p \to \text{Int} = \text{Int} \to 'w$ (Bool)

$'p = \text{Int}$

$\text{Int} = 'w$ (Bool)

$'p = 'w$

(3) ✓ ✗

(4) $'w \sim \text{Int}$