Subtyping Example (STFbR)

$\Gamma \vdash e : \tau$ typing relation

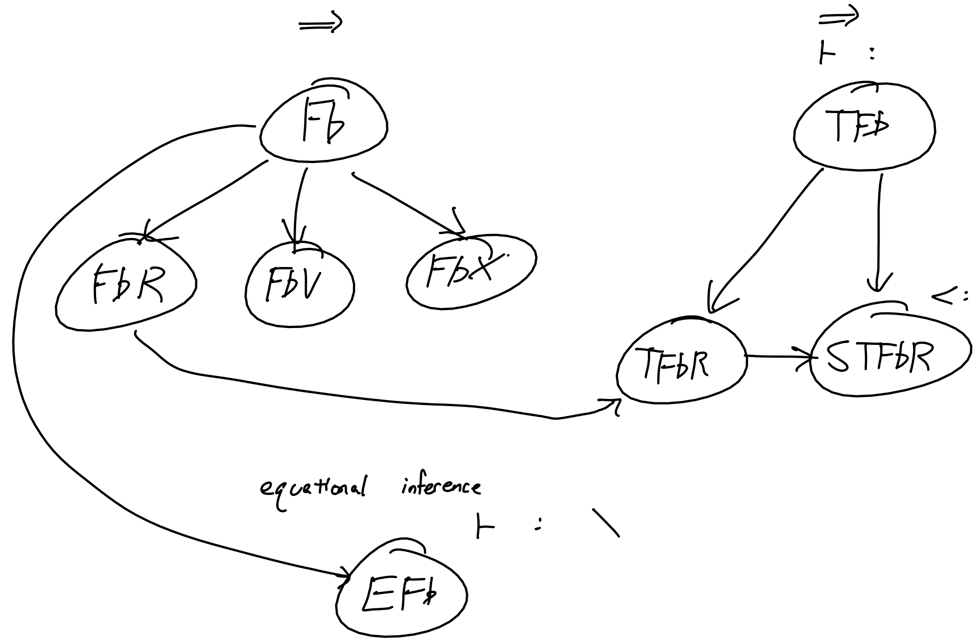
$\tau <: \tau$ subtyping relation

$\{a : Int\} \vdash a = a : Bool$

$\{a : Int; b : Bool\} <: \{b : Bool\}$

$$\text{Sub}\ \frac{\Gamma \vdash e : \tau \qquad \tau <: \tau'}{\Gamma \vdash e : \tau'} \qquad \text{App}\ \frac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'}$$

$$\text{App}\ \frac{\varnothing \vdash (\text{Function } w : \{a : Bool\} \to w) : \{a : Bool\} \to \{a : Bool\} \qquad \dfrac{\varnothing \vdash \{a = True; b = False\} : \tau \qquad \tau <: \{a : Bool\}}{\varnothing \vdash \{a = True; b = False\} : \{a : Bool\}}}{\varnothing \vdash \underbrace{(\text{Function } w : \{a : Bool\} \to w)}_{e_1}\ \underbrace{\{a = True; b = False\}}_{e_2} : \tau'}$$



Fb
FbR
FbV
FbX
TFb
TFbR
STFbR
EFb
equational inference

EFb

equational inference

$$e ::= \overbrace{\ldots}^{Fb}$$
$$v ::= \overbrace{\ldots}^{Fb}$$
$$\tau ::= Int \mid Bool \mid \tau \to \tau \mid \alpha \quad \text{types}$$
$$\alpha ::= 'a \mid 'b \mid \ldots \quad \text{type variables}$$
$$q ::= \tau = \tau$$
$$E ::= \{q, \ldots\}$$

$'b \setminus \{'a = Int,\ 'b = 'a\}$

$'b = Int$

EFb Type Inference Algorithm

1. Prove $\Gamma \vdash e : \tau \setminus E$ holds.
2. Deductive closure of E
3. Check inconsistencies
4. Type substitution

$Int \setminus \{'a = Int,\ 'b = 'a,\ 'b = Bool\}$

$'b = Int$

$Int = 'b$

$Int = Bool$

---

1. Type Derivation

$$\text{Int}\ \frac{}{\Gamma \vdash \mathbb{Z} : Int \setminus \varnothing}$$

$$\text{Not}\ \frac{\Gamma \vdash e : \tau \setminus E}{\Gamma \vdash Not\ e : Bool \setminus E \cup \{\tau = Bool\}}$$

$$\text{Plus}\ \frac{\Gamma \vdash e_1 : \tau_1 \setminus E_1 \qquad \Gamma \vdash e_2 : \tau_2 \setminus E_2}{\Gamma \vdash e_1 + e_2 : Int \setminus E_1 \cup E_2 \cup \{\tau_1 = Int, \tau_2 = Int\}}$$

$$\text{Function}\ \frac{\Gamma\{x : \alpha\} \vdash e : \tau \setminus E \qquad \alpha \text{ is fresh}}{\Gamma \vdash Function\ x \to e : \alpha \to \tau \setminus E}$$

$$\cfrac{\cfrac{\cfrac{}{\{a : 's\} \vdash a : 's \setminus \varnothing} \qquad \cfrac{}{\{a : 's\} \vdash 1 : Int \setminus \varnothing}}{\{a : 's\} \vdash a + 1 : Int \setminus \{'s = Int,\ Int = Int\}}}{\underbrace{\varnothing}_{\Gamma} \vdash Function\ \underbrace{a}_{x} \to \underbrace{a + 1}_{e} : 's \to Int \setminus \{'s = Int,\ Int = Int\}}$$

wanted

$Int \to Int$

$$\cfrac{\cfrac{}{\varnothing \vdash 1 : Int \setminus \varnothing} \qquad \cfrac{}{\varnothing : True : Bool \setminus \varnothing}}{\varnothing \vdash 1 + True : Int \setminus \{Int = Int,\ Bool = Int\}}$$

2. Deductive closure

* If $\tau_1 = \tau_2$ in E, then add $\tau_2 = \tau_1$.
* If $\tau_1 = \tau_2$ in E and $\tau_2 = \tau_3$ in E, then add $\tau_1 = \tau_3$.
* If $\tau_1 \to \tau_2 = \tau_1' \to \tau_2'$ in E, then add $\tau_1 = \tau_1'$ and $\tau_2 = \tau_2'$.