FbR

$$(\text{Function } r \to r.a) \quad \{a = 5\} \Rightarrow 5$$

$$(\text{Function } r \to r.a) \quad \{a = 5; b = 8\} \Rightarrow 5$$

Let $f = \text{Function } r \to r.a$ In

$$f \{a = 5\} + f \{a = 5; b = 8\}$$

$$\Rightarrow 10$$

TFbR

$$(\text{Function } r : \{a : Int\} \to r.a) \quad \{a = 5\} \Rightarrow 5$$

$$(\text{Function } r : \{a : Int; b : Int\} \to r.a) \quad \{a = 5; b = 8\} \Rightarrow 5$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 \; e_2 : \tau_2}$$

Let $f : \underline{\quad} =$

$$\text{Function } r : \underline{\quad} \to r.a$$

In

$$f \{a = 5\} + f \{a = 5; b = 8\}$$
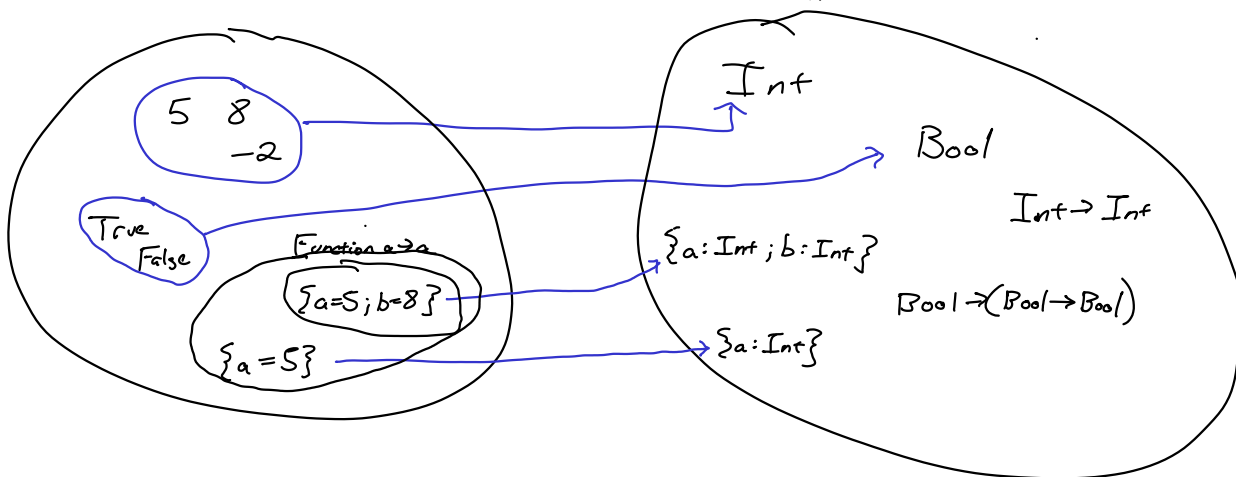

What is a type?

$\boxed{v : \tau} \longrightarrow$ what can I do (or not do) with this value?*

$\approx$ a type is a promise

$v \in \tau \longrightarrow$ a type is a set of values*

"a subtype is a subset of values"

all values   all types

Subtyping Relation

$$e \Rightarrow v$$
$$\langle S, e \rangle \Rightarrow \langle S, v \rangle$$
$$\Gamma \vdash e : \tau$$

$\leq$
$\subseteq$

Subtyping is a relation of the form $\quad \tau <: \tau$

$$\tau_1 <: \tau_2$$

TFbR: $\quad \tau ::= Int \mid Bool \mid \tau \rightarrow \tau \mid$
$$\{ l : \tau, ... \}$$

order doesn't matter

$$Int <: Int$$

$$\{ a : Int ; b : Bool \} <: \{ a : Int \} <: \{ \}$$
$$\tau \qquad\qquad \tau' \qquad \tau''$$

Reflexivity
$$\frac{}{\tau <: \tau}$$

Transitivity
$$\frac{\tau_1 <: \tau_2 \qquad \tau_2 <: \tau_3}{\tau_1 <: \tau_3}$$

~~**Symmetry**~~
$$\frac{\tau_1 <: \tau_2}{\tau_2 <: \tau_1}$$ (crossed out)

Record Subtyping (Width)
$$\frac{m > n}{\{ l_1 : \tau_1, ..., l_m : \tau_m \} <: \{ l_1 : \tau_1, ..., l_n : \tau_n \}}$$

Record Depth Subtyping
$$\frac{\tau_1 <: \tau_1' \qquad .... \qquad \tau_n <: \tau_n'}{\{ l_1 : \tau_1, ..., l_n : \tau_n \} <: \{ l_1 : \tau_1', ..., l_n : \tau_n' \}}$$

$$\{ a = \{ b = 8 ; c = True \} \} \qquad\qquad \{ a = \{ c = False \} \}$$

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Function Subtyping (in STFbR).

$$\{a:Int\} \longrightarrow \{p:Int\}$$

∀ I promise that you can pass me ANY record with an a label mapped to an Int.

I promise that I will only return records containing a p label mapped to an Int. ∃

opposite order
Contravariance

same order
covariance

Function Subtyping

$$\frac{\tau_1' <: \tau_1 \qquad \tau_2 <: \tau_2'}{\tau_1 \rightarrow \tau_2 <: \tau_1' \rightarrow \tau_2'}$$

$$\{\} \rightarrow \{p:Int; q:Bool\} \quad <: \quad \{a:Int\} \rightarrow \{p:Int\}$$

STFbR

$$\Gamma \vdash e : \tau$$

Subtype

$$\frac{\Gamma \vdash e : \tau_1 \qquad \tau_1 <: \tau_2}{\Gamma \vdash e : \tau_2}$$