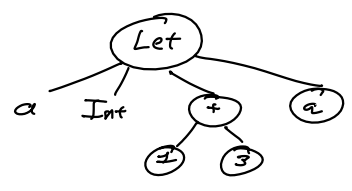


TFb

$e ::= \dots \mid \text{Let } x:\tau = e \text{ In } e$
 $v ::= \dots \mid \text{Function } x:\tau \rightarrow e$
 $\tau ::= \text{Int} \mid \text{Bool} \mid \tau \rightarrow \tau$
 $\Gamma ::= \{ x:\tau, \dots \}$

Let $a:\text{Int} = 1+3$ In a



Types are sets of values.*

$\Gamma \vdash e:\tau$

$e \Rightarrow v$

Assuming Γ , either $e \Rightarrow v$ s.t. $v \in \tau$ or e "runs forever".

e evaluates to v

Invariant: all free variables in e are mapped in Γ .

Invariant: e is closed.

$\emptyset \vdash 1:\text{Int}$	$\emptyset \vdash 3:\text{Int}$	
$\emptyset \vdash 1+3:\text{Int}$		$\{a:\text{Int}\} \vdash a:\text{Int}$
$\emptyset \vdash \text{Let } a:\text{Int} = 1+3 \text{ In } a$	$:\text{Int}$	

(Note: In the original image, green brackets underline the 'Int' type in the first row, the '1+3' expression in the second row, and the 'Int' type in the third row.)

Let $b:\text{Bool} = \text{True}$ In Let $a:\text{Int} = 1+3$ In a

	$\{b:\text{Bool}\} \vdash 1:\text{Int}$	$\{b:\text{Bool}\} \vdash 3:\text{Int}$
	$\{b:\text{Bool}\} \vdash 1+3:\text{Int}$	$\{b:\text{Bool}, a:\text{Int}\} \vdash a:\text{Int}$
$\emptyset \vdash \text{True}:\text{Bool}$	$\{b:\text{Bool}\} \vdash \text{Let } a:\text{Int} = 1+3 \text{ In } a : \text{Int}$	
$\emptyset \vdash \text{Let } b:\text{Bool} = \text{True} \text{ In } \dots$	$:\text{Int}$	

$\Gamma, x:\tau \vdash e:\tau'$
 $\Gamma \{x:\tau\} \vdash e:\tau'$

$\Gamma \vdash \text{Function } x:\tau \rightarrow e : \tau \rightarrow \tau'$

(Note: In the original image, green brackets underline 'e' and 'tau' in the expression.)

$(\text{Function } a \rightarrow a \ a) (\text{Function } a \rightarrow a \ a)$
 ω -combinator

$\Gamma \vdash e_1:\tau \rightarrow \tau' \quad \Gamma \vdash e_2:\tau$
 $\Gamma \vdash e_1 e_2:\tau'$

$(\text{Function } a: \tau \rightarrow a \ a) (\text{Function } a: \tau \rightarrow a \ a)$

Assuming TFb does not have Let Rec,
then TFb is normalizing
when evaluation is permitted only for
well-typed programs.

$\tau_\omega = \tau_\omega \rightarrow \tau_2$
 $= (\tau_\omega \rightarrow \tau_2) \rightarrow \tau_2$
 $= ((\tau_\omega \rightarrow \tau_2) \rightarrow \tau_2) \rightarrow \tau_2$
 \dots

TFbR

$e ::= \dots \mid \{l=e; \dots\} \mid e.l$
 $v ::= \dots \mid \{l=v; \dots\}$
 $\tau ::= \dots \mid \{l:\tau; \dots\}$

$\emptyset \vdash \{a=4; b=True\} : \{a:Int; b:Bool\}$

$\Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Gamma \vdash e_n : \tau_n$

$\Gamma \vdash \{l_1=e_1; \dots; l_n=e_n\} : \{l_1:\tau_1; \dots; l_n:\tau_n\}$

$\Gamma \vdash e : \{l_1:\tau_1; \dots; l_n:\tau_n\} \quad l=l_i$

$\Gamma \vdash e.l : \tau_i$

TFbS

$e ::= \dots \mid \text{Ref } e \mid !e \mid e := e$

$v ::= \dots \mid c$

$\tau ::= \dots \mid \text{Ref } \tau$

$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \text{Ref } e : \text{Ref } \tau}$

$\frac{\Gamma \vdash e : \text{Ref } \tau}{\Gamma \vdash !e : \tau}$

$\frac{\Gamma \vdash e_1 : \text{Ref } \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 := e_2 : \tau}$