TFb

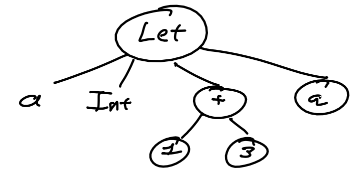
$e ::= \ldots \mid \text{Let } x:\tau = e \text{ In } e$

$v ::= \ldots \mid \text{Function } x:\tau \to e$

$\tau ::= \text{Int} \mid \text{Bool} \mid \tau \to \tau$

$\Gamma ::= \{x:\tau, \ldots\}$

Let $a:\text{Int} = 1+3$ In a



Types are sets of values.*

$\underline{\Gamma} \vdash \underline{e} : \underline{\tau}$

Assuming Γ, either $e \Rightarrow v$ s.t. $v \in \tau$ or e "runs forever".

Invariant: all free variables in e are mapped in Γ.

$\underline{e} \Rightarrow \underline{v}$

e evaluates to v

Invariant: e is closed.

$$\dfrac{\dfrac{\dfrac{}{\emptyset \vdash 1 : \text{Int}} \quad \dfrac{}{\emptyset \vdash 3 : \text{Int}}}{\emptyset \vdash 1+3 : \text{Int}} \qquad \dfrac{}{\{a:\text{Int}\} \vdash a : \text{Int}}}{\underbrace{\emptyset}_{\Gamma} \vdash \underbrace{\text{Let } a:\text{Int} = 1+3 \text{ In } a}_{e} : \underbrace{\text{Int}}_{\tau}}$$

Let $b:\text{Bool} = \text{True}$ In Let $a:\text{Int} = 1+3$ In a

$$\dfrac{\dfrac{}{\emptyset \vdash \text{True} : \text{Bool}} \qquad \dfrac{\dfrac{\dfrac{}{\{b:\text{Bool}\} \vdash 1 : \text{Int}} \quad \dfrac{}{\{b:\text{Bool}\} \vdash 3 : \text{Int}}}{\{b:\text{Bool}\} \vdash 1+3 : \text{Int}} \qquad \dfrac{}{\{b:\text{Bool}, a:\text{Int}\} \vdash a : \text{Int}}}{\{b:\text{Bool}\} \vdash \text{Let } a:\text{Int} = 1+3 \text{ In } a : \text{Int}}}{\emptyset \vdash \text{Let } b:\text{Bool} = \text{True In} \text{ - - - - - - - - - - - - -} : \text{Int}}$$

$\Gamma, x:\tau \vdash e : \tau'$

$$\dfrac{\Gamma \cup \{x:\tau\} \vdash e : \tau'}{\Gamma \vdash \underbrace{\text{Function } x:\tau \to e}_{e} : \underbrace{\tau \to \tau'}_{\tau}}$$

$$\dfrac{\Gamma \vdash e_1 : \tau \to \tau' \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\ e_2 : \tau'}$$

$\underbrace{(\text{Function } a \to a\ a)(\text{Function } a \to a\ a)}_{\omega\text{-combinator}}$

$(\text{Function } a:\_ \to a\ a)(\text{Function } a:\_ \to a\ a)$

$\tau_\omega = \tau_\omega \to \tau_2$

$= (\tau_\omega \to \tau_2) \to \tau_2$

$= ((\tau_\omega \to \tau_2) \to \tau_2) \to \tau_2$

$= \ldots$

Assuming TFb does not have Let Rec, then TFb is normalizing when evaluation is permitted only for well-typed programs.

TFbR

$$e ::= \cdots \mid \{\ell = e; \ldots\} \mid e.\ell$$
$$v ::= \cdots \mid \{\ell = v; \ldots\}$$
$$\tau ::= \cdots \mid \{\ell : \tau; \ldots\}$$

$$\emptyset \vdash \{a = 4; b = \mathrm{True}\} : \{a : \mathrm{Int}; b : \mathrm{Bool}\}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \cdots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash \{\ell_1 = e_1; \ldots; \ell_n = e_n\} : \{\ell_1 : \tau_1; \ldots; \ell_n : \tau_n\}}$$

$$\frac{\Gamma \vdash e : \{\ell_1 : \tau_1; \cdots; \ell_n : \tau_n\} \quad \ell = \ell_i}{\Gamma \vdash e.\ell : \tau_i}$$

$\boxed{\text{TFbS}}$

$$e ::= \cdots \mid \mathrm{Ref}\ e \mid !e \mid e := e$$
$$v ::= \cdots \mid c$$
$$\tau ::= \cdots \mid \mathrm{Ref}\ \tau$$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \mathrm{Ref}\ e : \mathrm{Ref}\ \tau} \qquad \frac{\Gamma \vdash e : \mathrm{Ref}\ \tau}{\Gamma \vdash\ !e : \tau} \qquad \frac{\Gamma \vdash e_1 : \mathrm{Ref}\ \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 := e_2 : \tau}$$