Types

A type system is a tool which produces an analysis of the behavior of a program before it is run.*

when?

what behavior?

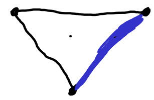
There is no perfect type system.

Complete

Call good programs "good"

Soundness

Call bad programs "bad".



Produces an answer in finite time

Decidability

Java, C, C++, TypeScript, OCaml "static" type system

Java, Python, JavaScript, Ruby, OCaml "dynamic" type system

C, C++ untyped

So far:

Grammar describes a set (usually of trees)

Inference rules to define a relation

$e \Rightarrow v$

- $e \Rightarrow v$ $\quad (\Rightarrow) \subseteq (e \times v)$

$1+1 \Rightarrow 2$ $\quad \langle 1+1, 2\rangle \in (\Rightarrow)$

$1+2 \not\Rightarrow 4$ $\quad \langle 1+2, 4\rangle \notin (\Rightarrow)$

- $\langle S, e\rangle \Rightarrow \langle S, v\rangle$ $\quad (\Rightarrow) \subseteq (S \times e \times S \times v)$

$\langle \emptyset, 1+1\rangle \Rightarrow \langle \emptyset, 2\rangle$

$\langle \{c_0 \mapsto 2\}, !c_0\rangle \Rightarrow \langle \{c_0 \mapsto 2\}, 2\rangle$

$\langle \{c_0 \mapsto 3\}, !c_0\rangle \not\Rightarrow \langle \emptyset, 3\rangle$

Now: another grammar, another relation.

$\langle S, e\rangle \Rightarrow \langle S, v\rangle$

TFb

$e ::= v \mid e + e \mid e - e \mid \cdots \mid \text{Let } x : \tau = e \text{ In } e$

$v ::= \mathbb{Z} \mid \text{True} \mid \text{False} \mid \text{Function } x : \tau \rightarrow e$

$\tau ::= \text{Int} \mid \text{Bool} \mid \tau \rightarrow \tau$

types as sets of values*

$$\frac{e_1 \Rightarrow v_1 \qquad e_2[v/x] \Rightarrow v_2}{\text{Let } x : \tau = e_1 \text{ In } e_2 \Rightarrow v_2}$$

$\text{Let } a : \text{Int} = 3+1 \text{ In } a = 4$

$\text{If True Then } 4 \text{ Else False} \Rightarrow 4$ will not typecheck

Typing relation

gamma

$\Gamma \vdash e : \tau$

$\Gamma ::= \{x : \tau, \ldots\}$

$$\frac{}{\Gamma \vdash \text{True} : \text{Bool}} \qquad \frac{}{\Gamma \vdash \text{False} : \text{Bool}} \qquad \frac{}{\Gamma \vdash \mathbb{Z} : \text{Int}}$$

$$\frac{\Gamma \vdash e_1 : \text{Int} \quad \Gamma \vdash e_2 : \text{Int}}{\Gamma \vdash e_1 + e_2 : \text{Int}} \qquad \frac{\Gamma \vdash e_1 : \text{Bool} \quad \Gamma \vdash e_2 : \text{Bool}}{\Gamma \vdash e_1 \text{ And } e_2 : \text{Bool}} \qquad \frac{\Gamma \vdash e_1 : \tau \quad \Gamma\{x : \tau\} \vdash e_2 : \tau'}{\Gamma \vdash \text{Let } x : \tau = e_1 \text{ In } e_2 : \tau'}$$

$$\frac{\Gamma \vdash e_1 : \text{Bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \text{If } e_1 \text{ Then } e_2 \text{ Else } e_3 : \tau} \qquad \frac{(x : \tau) \in \Gamma}{\Gamma \vdash x : \tau}$$