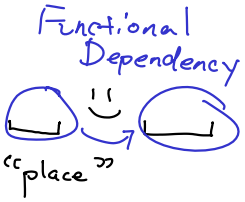
Relation

Given two sets S and T, a relation is a subset of $S \times T$.



$$\text{☺} = \{ 1,2 \;\; 2,3 \;\; 3,4 \;\; \vdots \}$$

$$\square < \square$$

$$< = \{ 1,2 \;\; 2,3 \;\; 1,3 \;\; 1,4 \;\; 2,4 \;\; \ldots \}$$

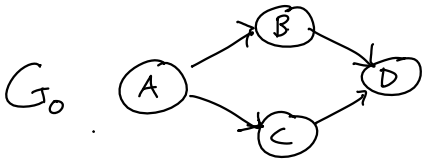
Define a relation on graphs

$\square \vdash \square \rightsquigarrow \square$

graph, vertex, vertex

A graph is a pair between a set of vertices V and a set of edges $E = \{\langle v, v \rangle, \ldots\}$.

$$\text{Reflexivity}\ \frac{v \in V}{\langle V,E \rangle \vdash v \rightsquigarrow v} \qquad \text{Step}\ \frac{\langle v_1, v_2 \rangle \in E \quad \langle V,E \rangle \vdash v_2 \rightsquigarrow v_3}{\langle V,E \rangle \vdash v_1 \rightsquigarrow v_3}$$



$$\text{Step}\ \frac{\langle A,C \rangle \in E_0 \qquad \text{Step}\ \dfrac{\langle C,D \rangle \in E_0 \qquad \text{Refl}\ \dfrac{}{\langle V_0,E_0 \rangle \vdash D \rightsquigarrow D}}{\langle V_0,E_0 \rangle \vdash C \rightsquigarrow D}}{\langle V_0,E_0 \rangle \vdash A \rightsquigarrow D}$$

$G_0 = \langle V_0, E_0 \rangle \qquad V_0 = \{A,B,C,D\}$

$$E_0 = \{ \langle A,B \rangle \;\; \langle A,C \rangle \;\; \langle B,D \rangle \;\; \langle C,D \rangle \}$$

Syntax of BOOL

int foo(int x) { return x;}
foo int)int return x x(}};

expression

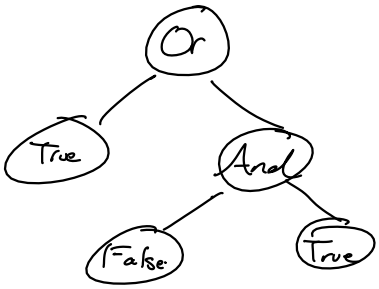
$e ::= v \mid e \text{ And } e \mid e \text{ Or } e \mid \text{Not } e \mid (e)$

values

$v ::= \text{True} \mid \text{False}$

] Concrete Syntax

True Or (False And True)



Operational Semantics

expression ⇒ value

$e \Rightarrow v$ (e evaluates to v)

$$\frac{}{v \Rightarrow v} \qquad \frac{e \Rightarrow \text{True}}{\text{Not } e \Rightarrow \text{False}} \qquad \frac{e \Rightarrow \text{False}}{\text{Not } e \Rightarrow \text{True}}$$

$$\frac{e_1 \Rightarrow \text{True} \quad e_2 \Rightarrow \text{True}}{e_1 \text{ And } e_2 \Rightarrow \text{True}} \qquad \frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1 \neq v_2}{e_1 \text{ And } e_2 \Rightarrow \text{False}}$$

$$\frac{e_1 \Rightarrow \text{False} \quad e_2 \Rightarrow \text{False}}{e_1 \text{ And } e_2 \Rightarrow \text{False}}$$

$$\frac{e_1 \Rightarrow \text{False} \quad e_2 \Rightarrow \text{False}}{\text{And} \Rightarrow \text{False}}$$

$$\frac{e_1 \Rightarrow \text{True} \quad e_2 \Rightarrow \text{True}}{e_1 \text{ Or } e_2 \Rightarrow \text{True}} \qquad \frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1 \neq v_2}{e_1 \text{ Or } e_2 \Rightarrow \text{True}}$$

$$\frac{e_1 \Rightarrow \text{False} \quad e_2 \Rightarrow \text{False}}{e_1 \text{ Or } e_2 \Rightarrow \text{False}}$$

An interpreter is a function that, given e, produces v such that $e \Rightarrow v$.

Properties of Operational Semantics

- An op. sem. is <u>deterministic</u> iff $\forall e.\ e \Rightarrow v_1$ and $e \Rightarrow v_2$ implies $v_1 = v_2$.
- An op. sem. is <u>normalizing</u> iff. $\forall e.\ \exists v.\ e \Rightarrow v$.

Properties of Expressions in an Operational Semantics

- An expression e <u>converges</u> iff $\exists v.\ e \Rightarrow v$.
- An expression e <u>diverges</u> iff it does not converge.