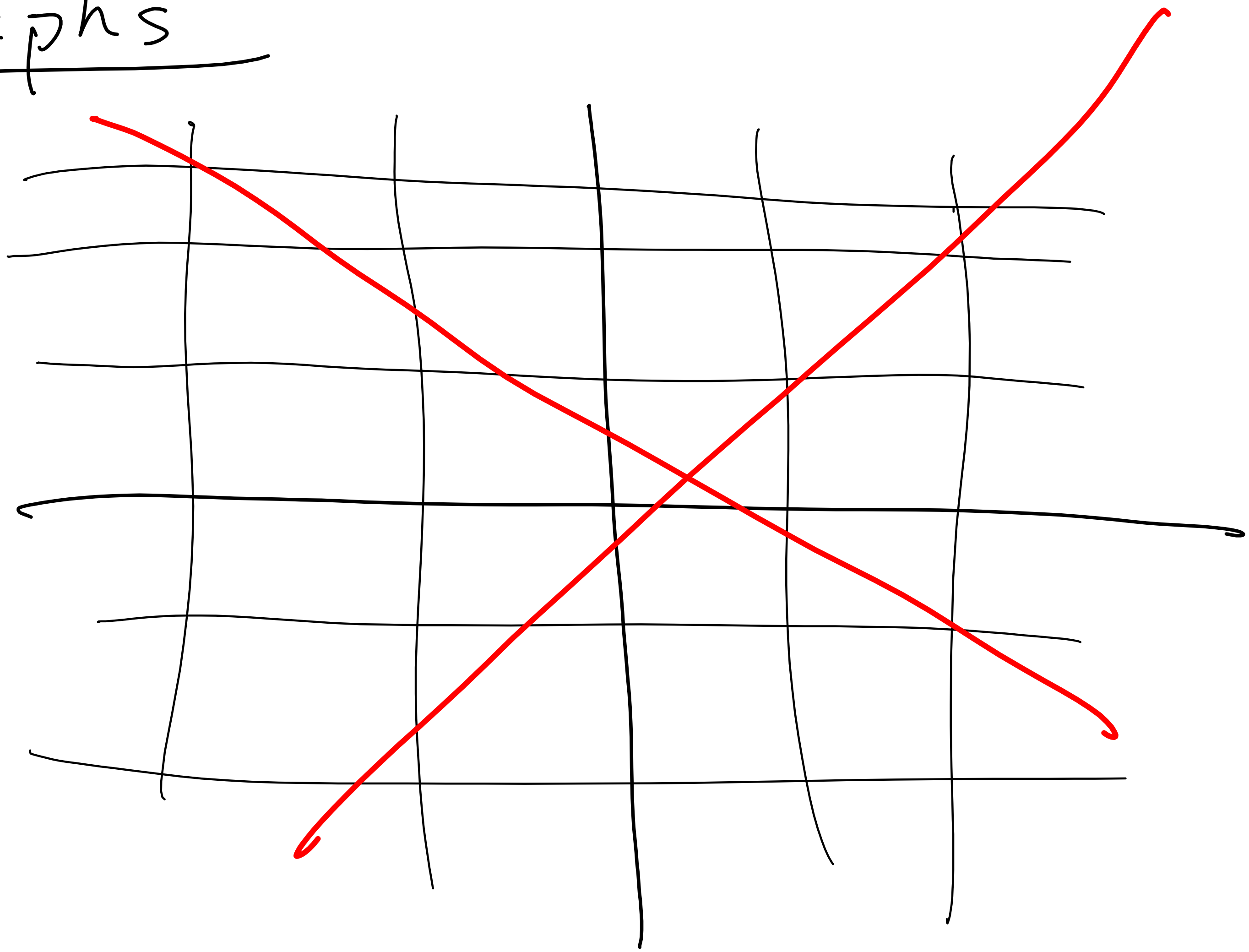
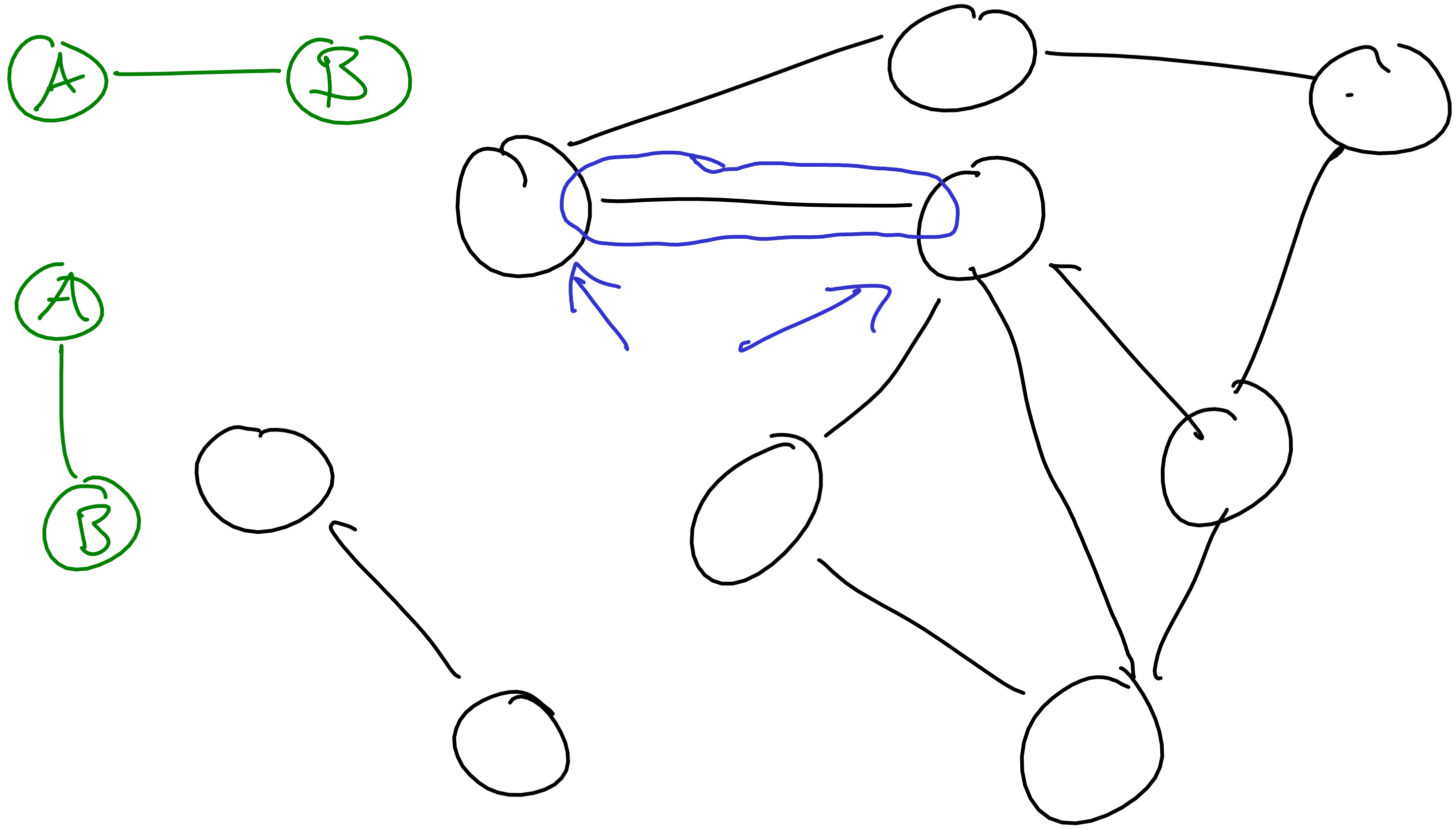
Graphs





<u>Graph</u> is a pair of two sets $\langle V, E \rangle$

V is a set of <u>vertices</u> (vertex)

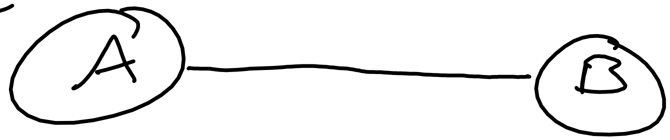
E is a set of <u>edges</u>

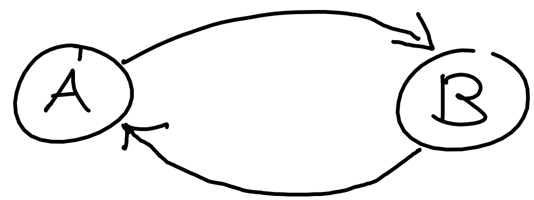
An <u>edge</u> is a pair of vertices

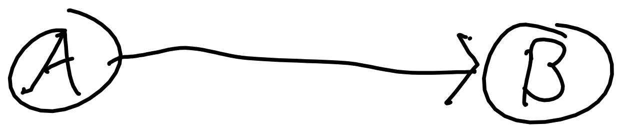
Undirected graph: if an edge exists ~~between~~ from v_1 and v_2, then an edge exists ~~between~~ from v_2 and v_1

source target

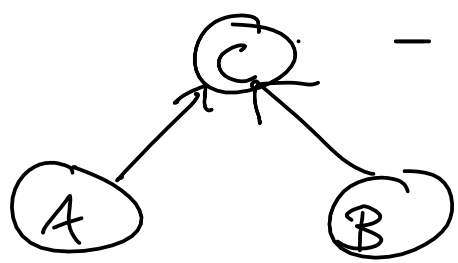
$$\langle v_1, v_2 \rangle \in E \implies \langle v_2, v_1 \rangle \in E$$





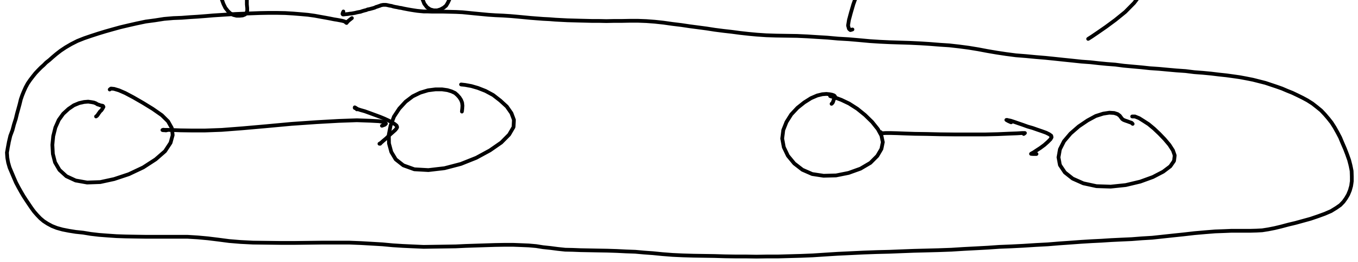


Directed graph makes no guarantee



connected

Connected graph where all nodes are touching via some sequence of edges (assuming edges are symmetric)



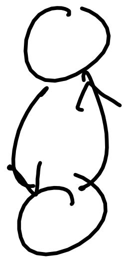
Unweighted (as above)

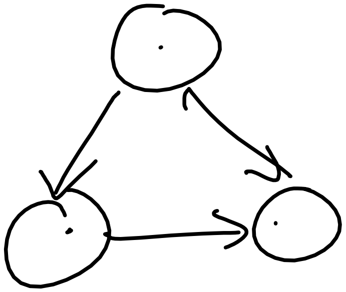
Weighted graph has 1 "weight" per edge

Edges E are of form $\langle V, V, W \rangle$

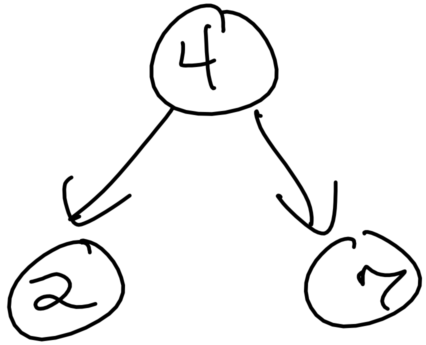
int, float

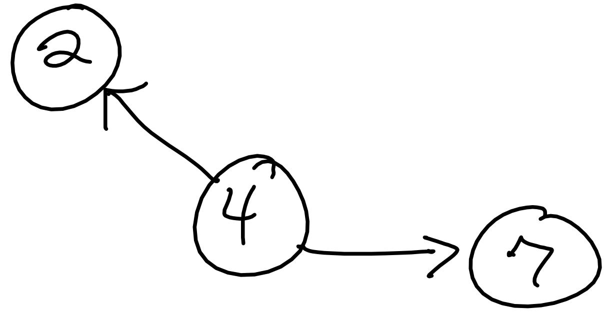
cyclic



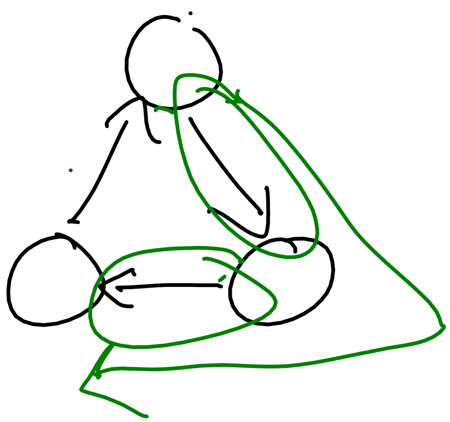


acyclic





<u>Tree</u> is a connected graph. which is directed and in which each node has at most <u>1</u> incoming edge and is <u>acyclic</u>



<u>Path</u> is a connected sequence of edges

A <u>cycle</u> is a path that stops where it started

<u>Acyclic</u>: no cycles anywhere

Undirected graph >0 edges is always cyclic

Directed graph <u>may</u> be cyclic

Directed acyclic graph (DAG)

Examples of graphs

Map

Travel schedule

weighted: time, cost

Social structure

nodes = people

edges: "work with" prefs

Vertices: things (movies, products)

Edges: predicted preferences (purchase history)

Algorithms on graphs

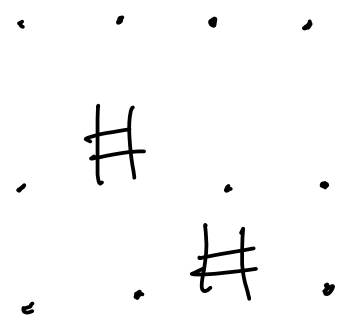
Planning a tour

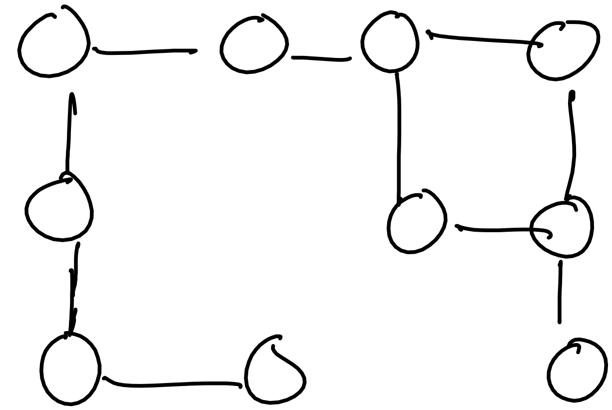
Plan my trip

Create groups

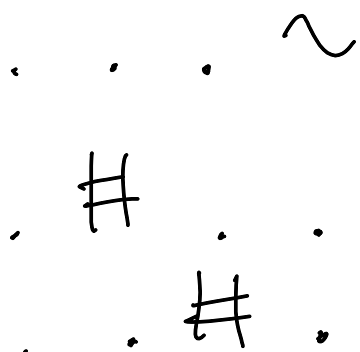
Recommendations

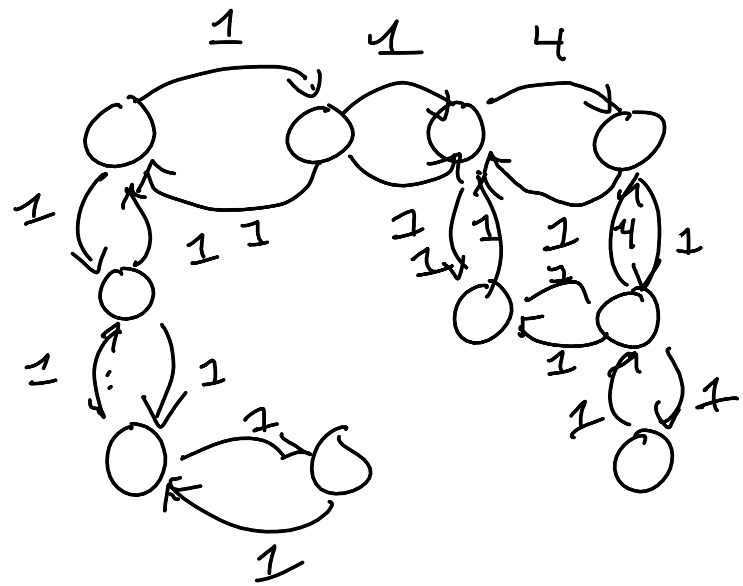
Lab 6



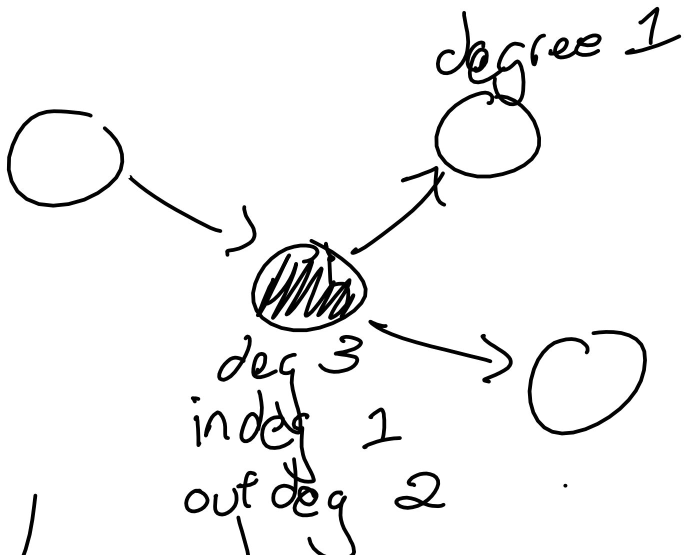


Lab 9





Degree of a vertex: # edges that touch it

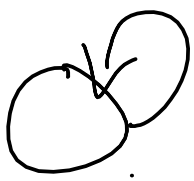


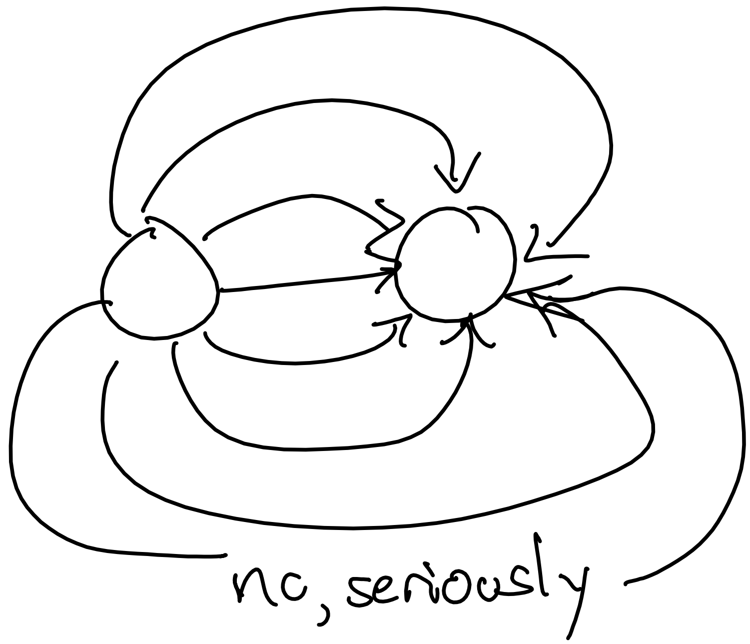
In-degree/out-degree:

v_1 and v_2 are **adjacent** if $\langle v_1, v_2 \rangle \in E$
or $\langle v_2, v_1 \rangle \in E$

Simple graph has no "self-loops" and no duplicate edges

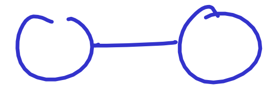
Not simple

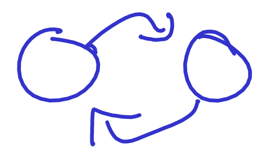




Max edges in an undirected simple graph?

directed ?

USG: $\frac{|V|\cdot(|V|-1)}{2}$ 

DSG: $|V|\cdot(|V|-1)$ 

$|V|$ n $|E|$ m

$$\sum_{v \in V} \text{degree}(v) = 2|E|$$