O(1):

~~is_sorted (A, n):~~

~~return False~~

incorrect

$$0+1+2+\cdots+n=\sum_{i=0}^{n} i=\frac{n(n+1)}{2}$$

$$P(n): 0+1+\cdots+n=\frac{n(n+1)}{2}$$

$$\boxed{\forall n\geq 0.\ P(n)}$$

$P(0)$ $\quad P(n)\Rightarrow P(n+1)$

$P(0)\Rightarrow P(1)\Rightarrow P(2)\Rightarrow P(3)\cdots$

$P(0): 0=\frac{0(0+1)}{2}=0$ ✓

$P(n)\Rightarrow P(n+1)$ $\quad 0+1+\cdots+n=\frac{n(n+1)}{2}$

$0+1+\cdots+n+1=\frac{(n+1)(n+2)}{2}$

$$0+1+\cdots+n+(n+1)=\frac{n(n+1)}{2}+n+1$$

$$=\frac{n(n+1)+2n+2}{2}$$

$$=\frac{n^2+n+2n+2}{2}$$

$$=\frac{n^2+3n+2}{2}$$

$$=\frac{(n+1)(n+2)}{2}$$

$$\forall_{x\geq 0} P(x): \sum_{i=0}^{x} 2^i = 2^{(x+1)} - 1$$

$$P(0): 1 = 2^0 = 2^1 - 1 = 2 - 1 = 1 \checkmark$$

$$P(n) \Rightarrow P(n+1): \overbrace{2^0 + \cdots + 2^n = 2^{n+1}}^{\text{Assume}} - 1$$

$$2^0 + \cdots 2^n + 2^{n+1} = 2^{n+1} - 1 + 2^{n+1}$$

$$= 2^{n+2} - 1$$

$$\forall n \geq 0. \quad \forall a > 1, a^0 + a^1 + \cdots + a^{\boxed{n}} = \frac{a^{\boxed{n+1}} - 1}{a - 1}$$

$P(0)$ $\quad$ $P(n) \Rightarrow P(n+1)$

$$P(0): a^0 = \frac{a^1 - 1}{a - 1} = 1 \checkmark$$

$$P(n) \Rightarrow P(n+1): a^0 + \cdots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

$$a^0 + \cdots a^n + a^{n+1} = \frac{a^{n+1} - 1}{a - 1} + a^{n+1}$$

$$a^0 + \cdots a^{\boxed{n+1}} = \frac{a^{n+1} - 1 + (a^{n+1})(a - 1)}{a - 1}$$

$$= \frac{\cancel{a^{n+1}} - 1 + a^{\boxed{n+2}} - \cancel{a^{n+1}}}{a - 1}$$

$$= \frac{a^{n+2} - 1}{a - 1}$$

Loop invariant

Function issorted(A, n):

For $\boxed{i} \leftarrow 0 \,..\, n-2$:

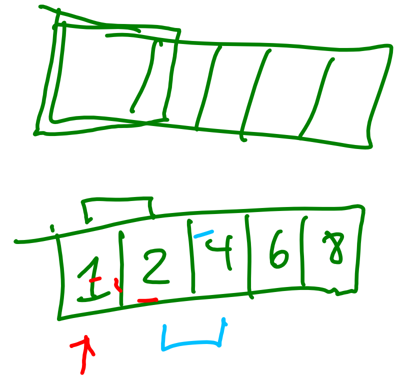
If $A[i] > A[i+1]$:

Return False

Return True.

The first i elements are sorted, w.r.t. both neighbors

The first i+1 elements are in order w.r.t. everything to the left!



$S(n)$: The first n+1 elements are in order wrt the element to their left.

$\underbrace{S(0)}$ ✓ $\qquad S(n) \Rightarrow S(n+1)$

$A[i] \leq A[i+1] \qquad S(n) \qquad \boxed{A[n] \leq A[n+1]}$

$O(n^3)$

$2n^2+n$ is $O(n^3)$

$\exists c>0, k\geq 1.\ \forall n\geq k.\ c(2n^2+n)\leq n^3$

$$\frac{2n^2+n}{4}\leq n^3$$

$2n^2+\leq 4n^3$

$2n^2+n\leq 4n^3$

$n^2+n^2+n\leq n^3+n^3+n^3+n^3$ $n^2+n\leq 3n^3$

$n\leq 2n^3$