```
bool isSorted(int count, int* array) {
    return false;
}
```

wrong

$\}\ O(1)$

Correctness?

Mathematical Induction

$$0+1+2+3+\cdots+k = \sum_{i=0}^{k} i = \frac{k(k+1)}{2}$$

$$P(n) \equiv \boxed{\sum_{i=0}^{n} i = \frac{n(n+1)}{2}}$$

natural number: $0, 1, \ldots$

$\mathbb{N}$

$$1 = 1 = \frac{1(1+1)}{2} = \frac{1 \cdot 2}{2} = 1$$

$$1+2 = 3 = \frac{2(2+1)}{2} = \frac{6}{2} = 3$$

$$1+2+3 = 6$$

$$1+2+3+4 = 10$$

- Base case: $P(0)$
- Inductive case: $P(n) \Longrightarrow P(n+1)$

$$P(0) \equiv \sum_{i=0}^{0} i = \frac{0(0+1)}{2}$$

$$0 = \frac{0}{2} \quad \checkmark$$

$$P(n) \Rightarrow P(n+1)$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2} \Longrightarrow \sum_{i=0}^{n+1} = \frac{(n+1)(n+2)}{2}$$

$$0+1+2+\cdots+n+(n+1) = \frac{(n+1)(n+2)}{2}$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$0+1+2+\cdots+n = \frac{n(n+1)}{2}$$

$$0+1+2+\cdots+n+(n+1) = \frac{n(n+1)}{2} + n + 1$$

$$\sum_{i=0}^{n+1} = \frac{n(n+1)}{2} + n + 1$$

$$= \frac{n^2+n}{2} + (n+1) = \frac{n^2+n}{2} + \frac{2n+2}{2} = \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2}$$

$$= \frac{n(n+1)}{2} + (n+1) = \frac{n(n+1)}{2} + \frac{2(n+1)}{2} = n\frac{(n+1)}{2} + 2\frac{(n+1)}{2}$$

$$= (n+2)\frac{(n+1)}{2}$$

$P(0)$ $\quad P(1)\checkmark \quad P(2)\checkmark \quad P(3)\checkmark \quad \ldots\ldots$

$P(n) \Rightarrow P(n+1)$

Prove by induction:

$$P(n) \equiv \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

$$2^0 = 2^1 - 1$$
$$2^0 + 2^1 + 2^2 = 2^3 - 1$$
$$1 + 2 + 4 = 8 - 1$$

Base case:

$$P(0): \sum_{i=0}^{0} 2^i = 2^{0+1} - 1$$

$$2^0 = 2^1 - 1$$
$$1 = 2 - 1 \checkmark$$

Inductive case:

$$P(n) \Rightarrow P(n+1)$$

$$\underbrace{\sum_{i=0}^{n} 2^i = 2^{n+1} - 1} \Rightarrow \sum_{i=0}^{n+1} 2^i = 2^{n+2} - 1$$

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

$$2^3 = 2 \cdot 2 \cdot 2 = 2^2 \cdot 2$$

$$2^0 + 2^1 + \dots + 2^n = 2^{n+1} - 1$$
$$2^0 + 2^1 + \dots + 2^n + 2^{n+1} = 2^{n+1} - 1 + 2^{n+1}$$
$$\sum_{i=0}^{n+1} 2^i = 2^{n+1} - 1 + 2^{n+1} = 2 \cdot 2^{n+1} - 1 = 2^{n+2} - 1$$

Prove by induction:

For any value $a > 1$..

$$P(n) \equiv \quad a^1 + a^2 + \dots + a^n = \frac{a(a^n - 1)}{a-1}$$

Base case:

$$P(1) \equiv \sum_{i=1}^{1} a^i = \frac{a(a^1-1)}{a-1}$$

$$a^1 = \frac{a\cancel{(a-1)}}{\cancel{a-1}}$$

$$a = a$$

$$\frac{x-1}{x-1} = 1$$

if $x=1$

$$\frac{0}{0} \neq 1$$

Inductive case:

$$P(n) \Rightarrow P(n+1)$$

$$\sum_{i=1}^{n} a^i = \frac{a(a^n-1)}{a-1} \implies \sum_{i=1}^{n+1} a^i = \frac{a(a^{n+1}-1)}{a-1}$$

$$\sum_{i=1}^{n+1} a^i = \frac{a(a^n-1)}{a-1} + a^{n+1}$$

$$= \frac{a(a^n-1)}{a-1} + \frac{a^{n+1}(a-1)}{a-1}$$

$$= \frac{a^{n+1}-a}{a-1} + \frac{a^{n+2}-a^{n+1}}{a-1} = \frac{a^{n+2}-a}{a-1} = \frac{a(a^{n+1}-1)}{a-1}$$

☆ !! QED!

```
int summate(int x) {
    if (x==0) {
        return 0;
    } else {
        return x + summate(x-1);
    }
}
```

$P(n) \equiv$

Prove by induction that $\boxed{\text{summate}(n) \text{ is } \sum_{i=1}^{n} i}$

Base case:

$$P(0) \equiv \text{summate}(0) \text{ is } \sum_{i=1}^{0} i = 0$$

$$P(n) \Rightarrow P(n+1) \equiv \text{If summate}(n) \text{ is } \sum_{i=1}^{n} i \text{ then summate}(n+1) \text{ is } \sum_{i=1}^{n+1} i.$$

Object invariants

$P(n) \equiv$ After n method calls, my object invariants are true.

$P(0) \equiv$ After 0 method calls .....

After the constructor

$P(n) \Longrightarrow P(n+1) \equiv$ .....

During method call