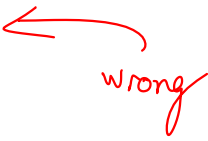


```
bool isSorted(int count, int* array) {  
    return false;   
}
```

} $O(1)$

Correctness?

Mathematical Induction

$$0+1+2+3+\dots+k = \sum_{i=0}^k i = \frac{k(k+1)}{2}$$

$$P(n) \equiv \sum_{i=0}^n i = \frac{n(n+1)}{2}$$

natural number: 0, 1, ...
 \mathbb{N}

$$1 = 1 = \frac{1(1+1)}{2} = \frac{1 \cdot 2}{2} = 1$$

$$1+2 = 3 = \frac{2(2+1)}{2} = \frac{6}{2} = 3$$

$$1+2+3 = 6$$

$$1+2+3+4 = 10$$

• Base case: $P(0)$

• Inductive case: $P(n) \implies P(n+1)$

$$P(0) \equiv \sum_{i=0}^0 i = \frac{0(0+1)}{2}$$

$$0 = \frac{0}{2} \quad \checkmark$$

$P(n) \implies P(n+1)$

$$\sum_{i=0}^n i = \frac{n(n+1)}{2} \implies \sum_{i=0}^{n+1} i = \frac{(n+1)(n+2)}{2}$$

$$0+1+2+\dots+n+(n+1) = \frac{(n+1)(n+2)}{2}$$

$$\sum_{i=0}^n i = \frac{n(n+1)}{2}$$

$$0+1+2+\dots+n = \frac{n(n+1)}{2}$$

$$0+1+2+\dots+n+(n+1) = \frac{n(n+1)}{2} + n+1$$

$$\sum_{i=0}^{n+1} i = \frac{n(n+1)}{2} + n+1$$

$$= \frac{n^2+n}{2} + (n+1) = \frac{n^2+n}{2} + \frac{2n+2}{2} = \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2}$$

$$= \frac{n(n+1)}{2} + (n+1) = \frac{n(n+1)}{2} + \frac{2(n+1)}{2} = n \frac{(n+1)}{2} + 2 \frac{(n+1)}{2}$$

$$= (n+2) \frac{(n+1)}{2}$$

$P(0)$ $P(1) \checkmark$ $P(2) \checkmark$ $P(3) \checkmark$

$P(n) \implies P(n+1)$

Prove by induction:

$$P(n) \equiv \sum_{i=0}^n 2^i = 2^{n+1} - 1$$

$$2^0 = 2^1 - 1$$
$$2^0 + 2^1 + 2^2 = 2^3 - 1$$
$$1 + 2 + 4 = 8 - 1$$

Base case:

$$P(0) : \sum_{i=0}^0 2^i = 2^{0+1} - 1$$

$$2^0 = 2^1 - 1$$
$$1 = 2 - 1 \quad \checkmark$$

Inductive case:

$$P(n) \Rightarrow P(n+1)$$

$$\underbrace{\sum_{i=0}^n 2^i = 2^{n+1} - 1}_{\text{inductive hypothesis}} \Rightarrow \sum_{i=0}^{n+1} 2^i = 2^{n+2} - 1$$

$$\sum_{i=0}^n 2^i = 2^{n+1} - 1$$

$$2^3 = 2 \cdot 2 \cdot 2 = 2^2 \cdot 2$$

$$2^0 + 2^1 + \dots + 2^n = 2^{n+1} - 1$$

$$2^0 + 2^1 + \dots + 2^n + 2^{n+1} = 2^{n+1} - 1 + 2^{n+1}$$

$$\sum_{i=0}^{n+1} 2^i = 2^{n+1} - 1 + 2^{n+1} = 2 \cdot 2^{n+1} - 1 = 2^{n+2} - 1$$

Prove by induction:

For any value $a > 1$.

$$P(n) \equiv a^1 + a^2 + \dots + a^n = \frac{a(a^n - 1)}{a - 1}$$

Base case:

$$P(1) \equiv \sum_{i=1}^1 a^i = \frac{a(a^1 - 1)}{a - 1}$$

$$a^1 = \frac{a(a - 1)}{a - 1}$$

$$a = a$$

$$\frac{x-1}{x-1} = 1$$

if $x=1$

$$\frac{0}{0} \neq 1$$

Inductive case:

$$P(n) \Rightarrow P(n+1)$$

$$\sum_{i=1}^n a^i = \frac{a(a^n - 1)}{a - 1} \Rightarrow \sum_{i=1}^{n+1} a^i = \frac{a(a^{n+1} - 1)}{a - 1}$$

$$\sum_{i=1}^{n+1} a^i = \frac{a(a^n - 1)}{a - 1} + a^{n+1}$$

$$= \frac{a(a^n - 1)}{a - 1} + \frac{a^{n+1}(a - 1)}{a - 1}$$

$$= \frac{a^{n+1} - a}{a - 1} + \frac{a^{n+2} - a^{n+1}}{a - 1} = \frac{a^{n+2} - a}{a - 1} = \frac{a(a^{n+1} - 1)}{a - 1}$$

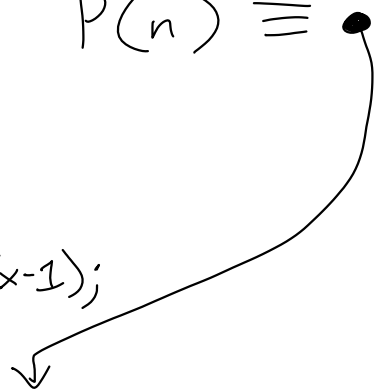
★ !!
QED!

```

int summate(int x) {
  if (x == 0) {
    return 0;
  } else {
    return x + summate(x-1);
  }
}

```

$P(n) \equiv$



summate(n) is $\sum_{i=1}^n i$

Prove by induction that

Base case:

$P(0) \equiv \text{summate}(0) \text{ is } \sum_{i=1}^0 i = 0$

$P(n) \Rightarrow P(n+1) \equiv \text{IF summate}(n) \text{ is } \sum_{i=1}^n i \text{ then summate}(n+1) \text{ is } \sum_{i=1}^{n+1} i.$

Object invariants

$P(n) \equiv$ After n method calls, my
object invariants are true.

$P(0) \equiv$ After 0 method calls
After the constructor

$P(n) \Rightarrow P(n+1) \equiv$
During method call