

	0	1	2	3
		5	10	9
		q	z	a
x		✓	✓	✓

K  
V  
Valid

$5^k \mapsto v$   
 $5^0 4 = \underline{1}$

$10 \mapsto z$   
 $9 \mapsto a$

	0	1	2	3	4	5
		1	7	3	13	
		q	a	z	b	
		✓	✓	✓	✓	

amortized worst-case  
expected

worst-case  
on average  
over a series of calls

inserts	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
copies	0	0	2	0	4	0	0	0	8	0	0	0	0	0	0	0

average case

uniformly random / evenly weighted

best-case

expected worst-case

data  
evil

Universe  
indifferent

worst-case

evil

evil

Function  $EWC(n)$ :

$i \leftarrow 0$

while  $i < n$ :

$j \leftarrow \text{random} \#$ : either 0 or  $\frac{n}{2}$

$i \leftarrow j$

$WC = O(n)$

$EWC = O(1)$

# Heaps

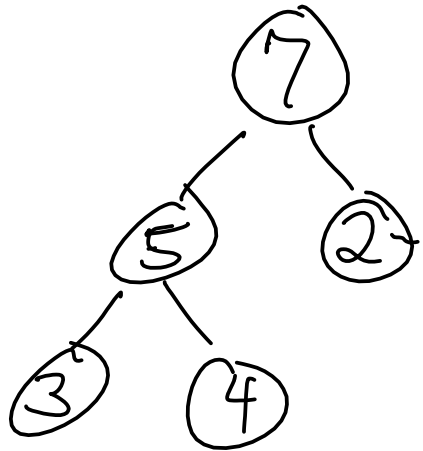
enqueue/dequeue:  $O(\log n)$

complete

children  $\leq$  parent

Heapify

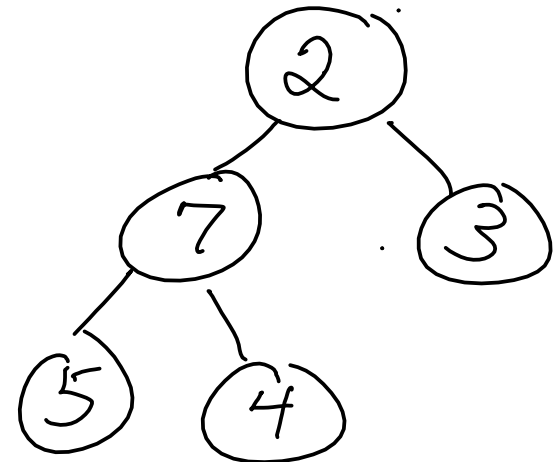
7 5 2 3 4



heapify:

for each node from  
bottom to top:  
bubble down

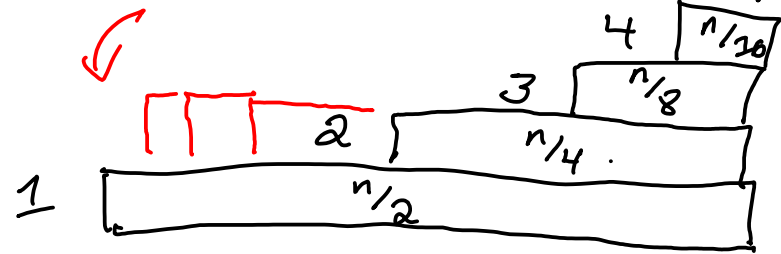
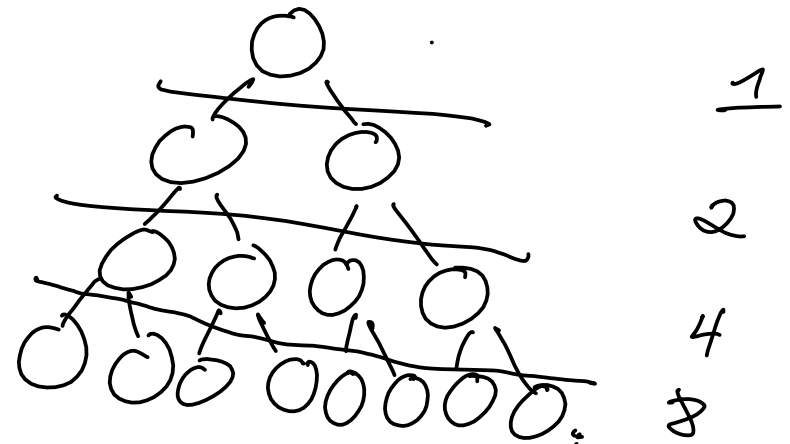
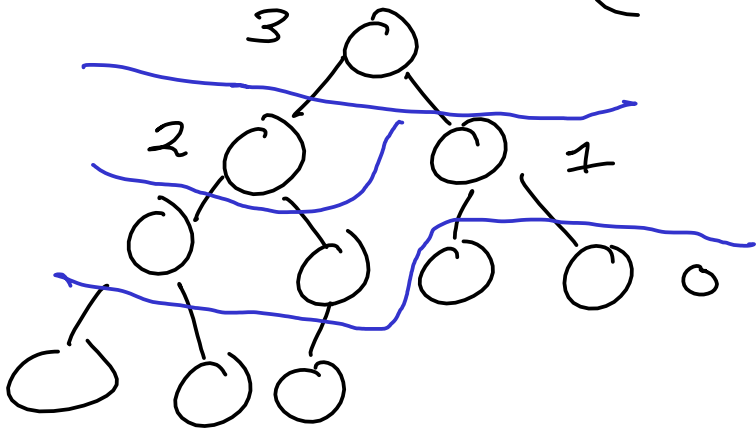
2 7 3 5 4



$$bd: O(\log n)$$

$$\times \text{ each node: } O(n^i)$$

$$O(n \log n)$$



$$8 \cdot 0 + 4 \cdot 1 + 2 \cdot 2 + 1 \cdot 3$$

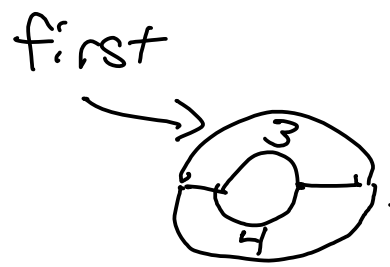
$$\frac{2^n}{2} \cdot 0 + \frac{n}{4} \cdot 1 + \dots + 2 \cdot (\log_2 n - 1) + 1 \cdot \log_2 n$$

$$\sum_{i=0}^{\log_2 n} \frac{n}{2^{i+1}} \cdot i = \frac{n}{2} \cdot \sum_{i=0}^{\log_2 n} \frac{i}{2^i} < \frac{n}{2} \sum_{i=0}^{\infty} \frac{i}{2^i} = \frac{n}{2} \cdot 2 = n$$

$$\frac{n}{2} \cdot 2 = n$$

# Cycle List : ADT

T get (int index)  
void insert At Head (T)  
T removeHead ()  
void rotate Right ()



get(0)  $\Rightarrow$  3  
get(1)  $\Rightarrow$  4  
get(2)  $\Rightarrow$  3