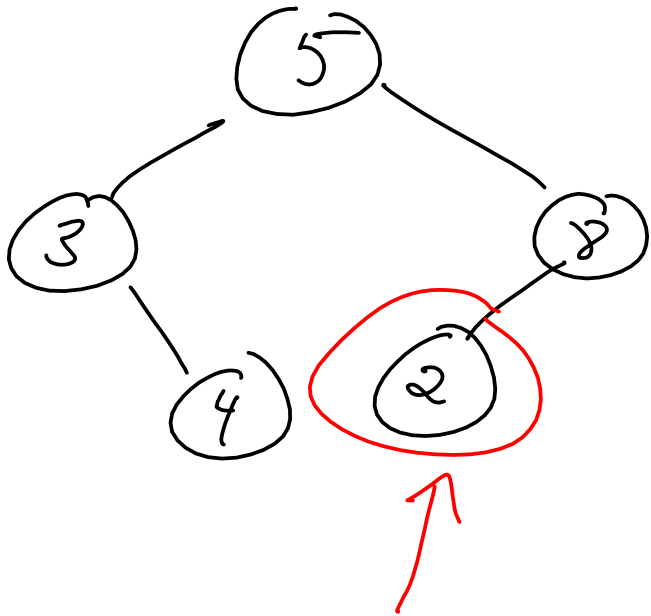
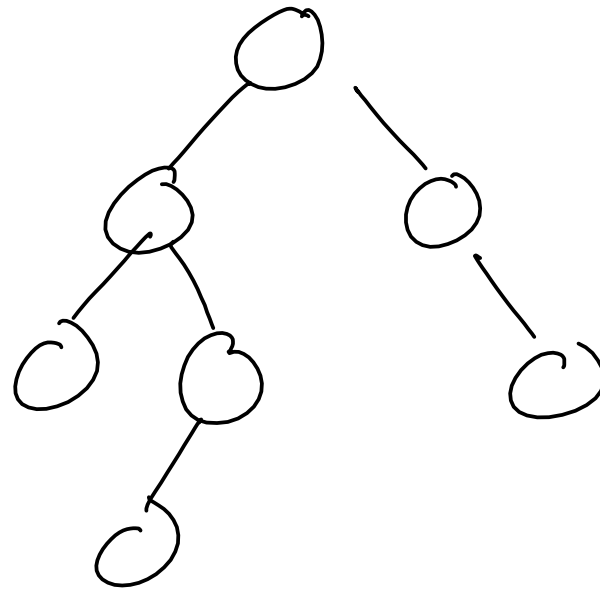


Binary Tree - each node has at most 2 children

BST - binary tree where each node's left subtree has data $<$ that node likewise for right

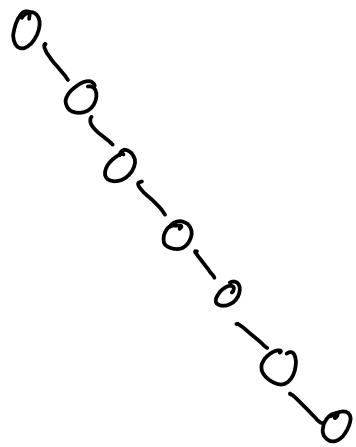
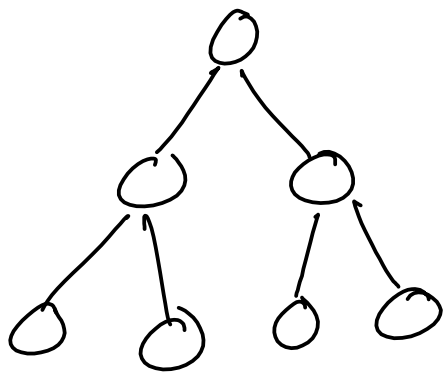


breaks BST
invariant



height - # edges between root and furthest leaf

Balancing BSTs



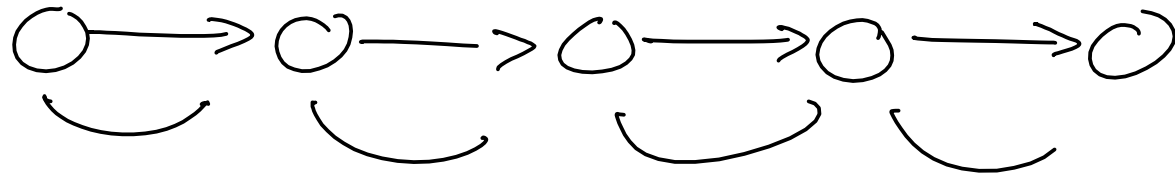
get
insert
update
remove



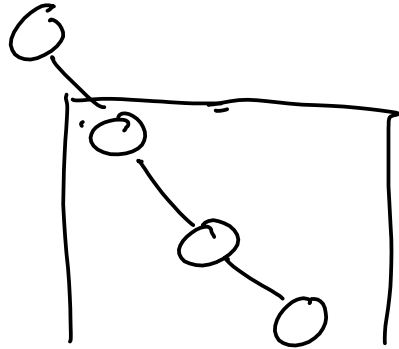
~~$O(\log n)$~~

$O(\text{height})$

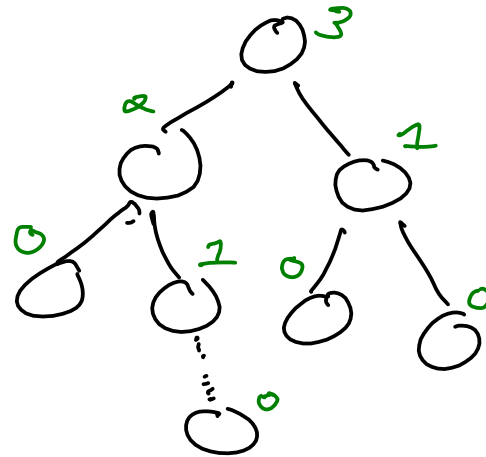
$O(n)$



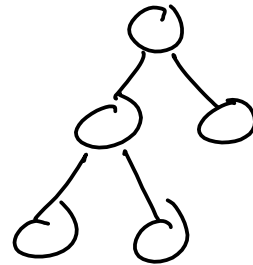
size w/o a field
is $O(n)$



height
 $O(n)$



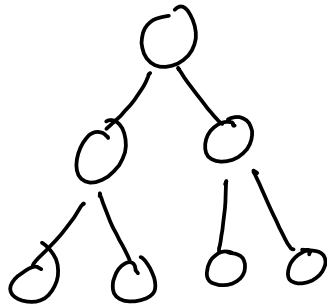
Complete



Level 0

Full

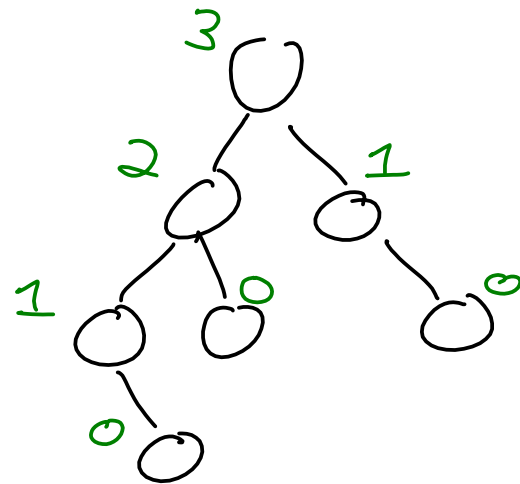
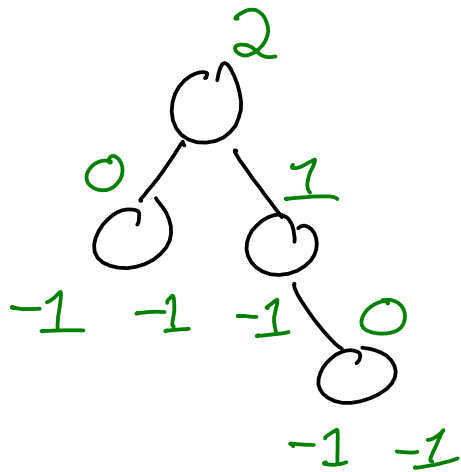
Level 1



AVL Tree is a BST w/ invariant

Adelson-Velsky
Landis
1962

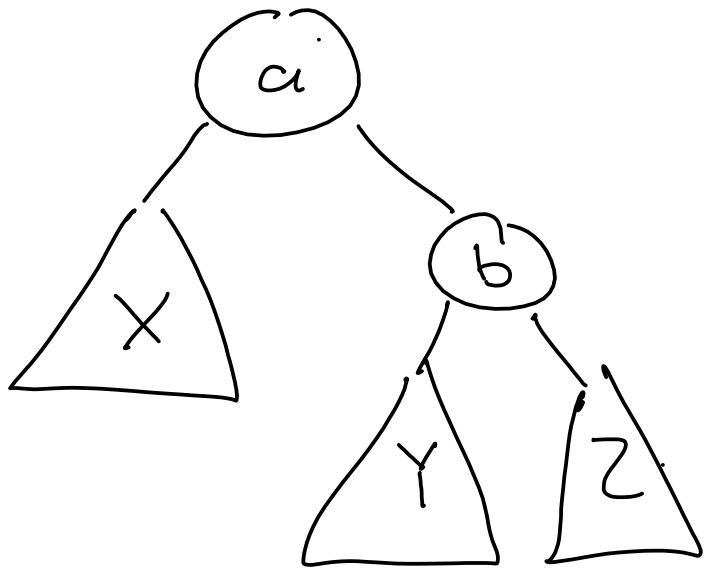
Every node has left subtree
& right subtree which differ
in height by at most 1



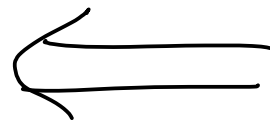
Tree Rotation

$$X < a$$
$$Z > b$$

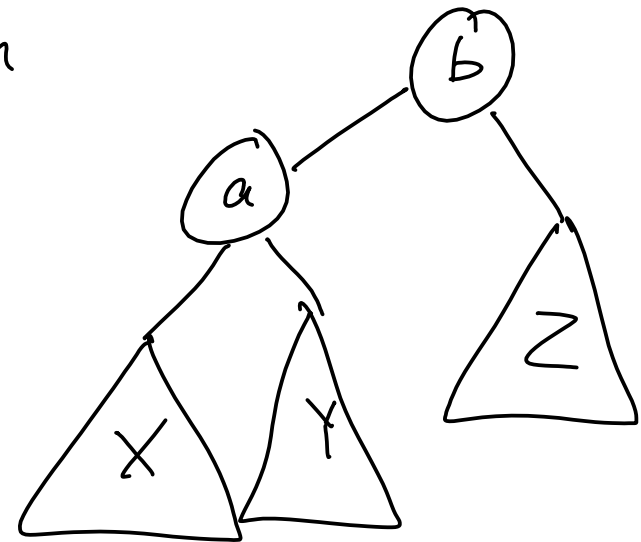
$$a < Y < b$$



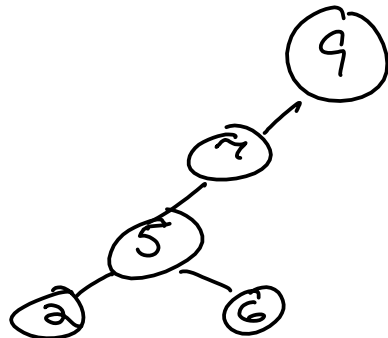
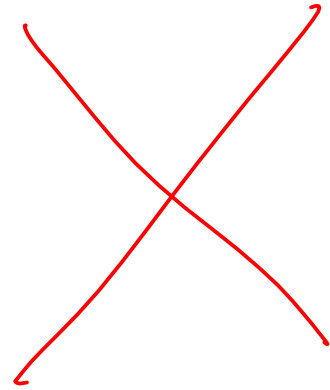
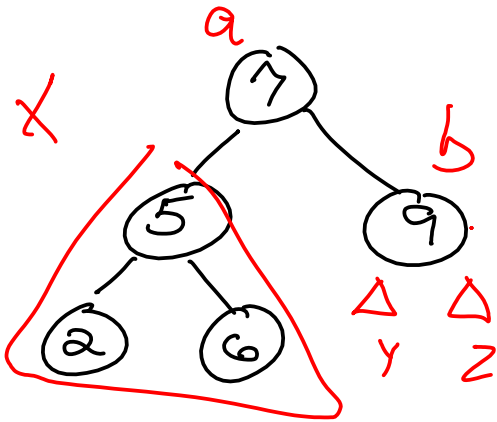
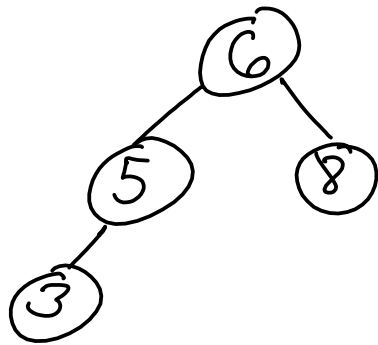
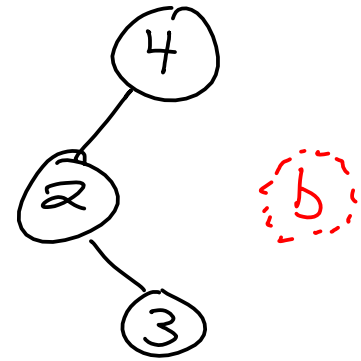
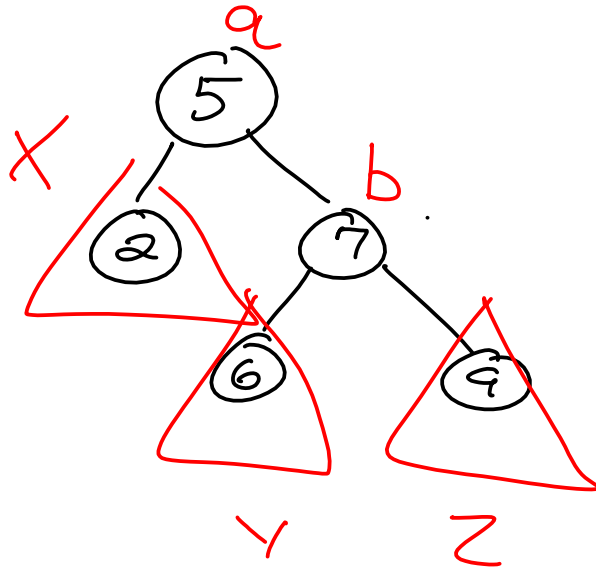
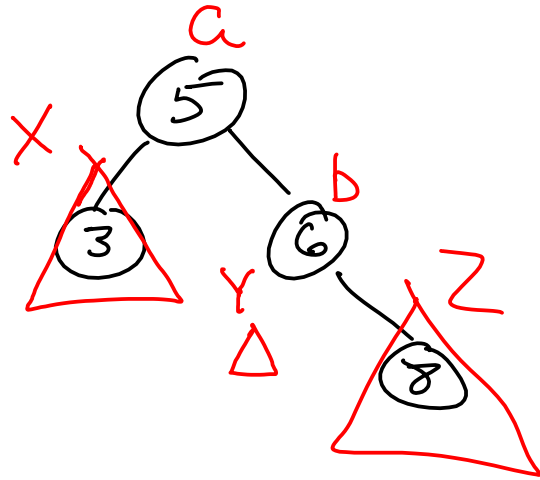
Left Rotation



Right Rotation



Left-rotate



AVL insert

Recompute height
Some Rotations?

Function $\text{insert}(\text{Node node}, k \text{ key}, v \text{ value}) :$

IF node is empty tree:

Return new Node(key, value)

IF $\text{key} > \text{node} \rightarrow \text{key} :$

$\text{node} \rightarrow \text{right} \leftarrow \text{insert}(\text{node} \rightarrow \text{right}, \text{key}, \text{value})$
Return node

Else IF $\text{key} < \text{node} \rightarrow \text{key} :$

$\text{node} \rightarrow \text{left} \leftarrow \text{insert}(\text{node} \rightarrow \text{left}, \text{key}, \text{value})$
Return node

Else:

throw exception! $\frac{1}{2}$