

$O(n^2)$

↙ size = n

Function is\_sorted(array)

$n$  times → for each  $i$  in  $0 \dots \text{size}(\text{array}) - 1$ :

$n$  times  
 $n-1$  times → for each  $j$  in  $i+1 \dots \text{size}(\text{array}) - 1$ :

1 → if array[i] > array[j]

1 → return false

end if

end for

end for

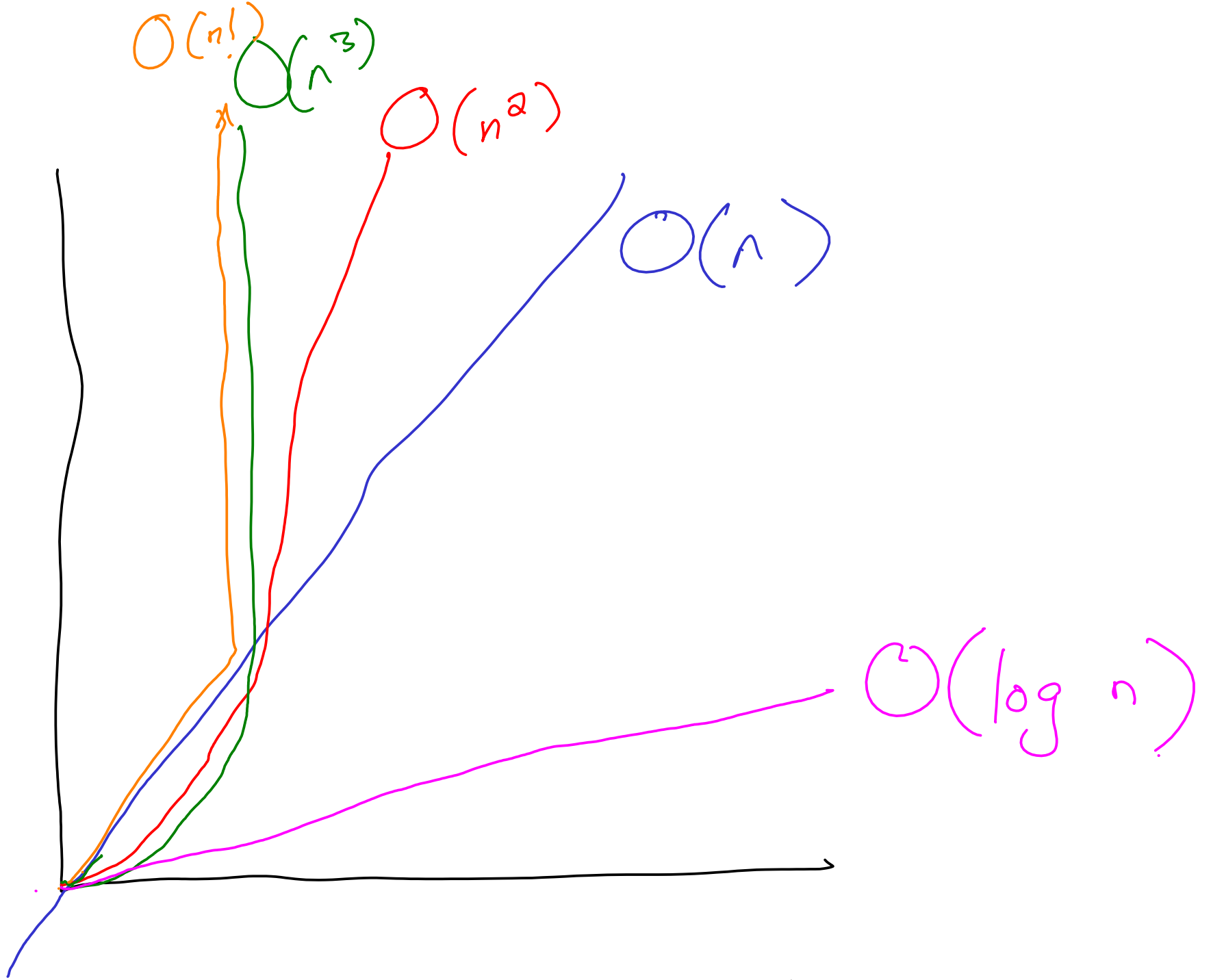
1 → return true

End Function

$(n + n-1 + n-2 \dots + 1)$

$n(n-1)$

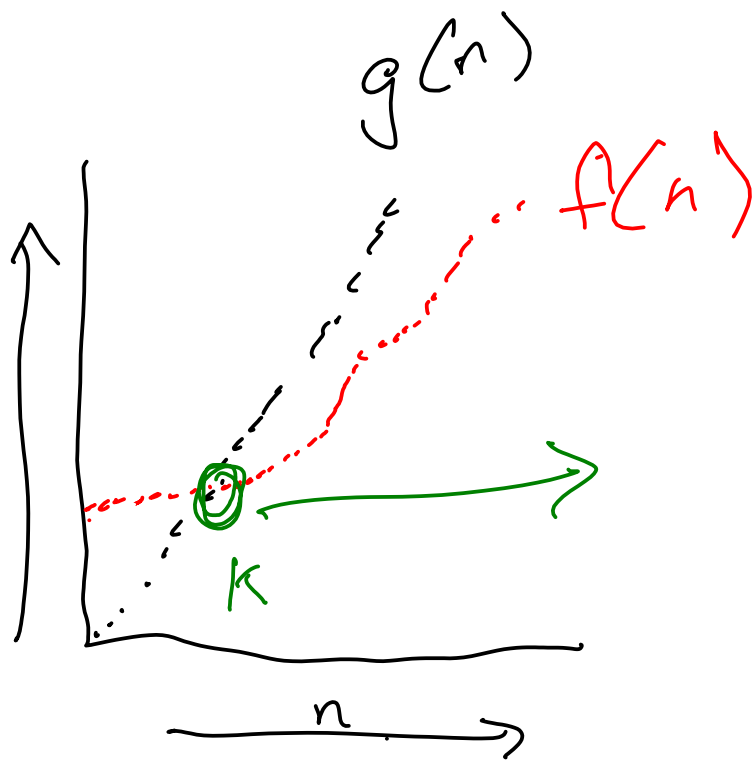
$(n^2 - n + 1) / 2$



$f(n)$  is  $O(g(n))$

iff

$\exists c > 0, k \geq 1. \forall n \geq k. f(n) \leq cg(n)$



Let  $f(n) = n^2$ . Then  $f(n)$  is  $O(n^3)$ .

$$\exists c > 0, k \geq 1. \forall n \geq k. n^2 \leq cn^3$$

Let  $c = 1, k = 1$ . Then it suffices to show

$$\forall n \geq 1. n^2 \leq n^3$$

$$\forall n \geq 1. 1 \leq n$$

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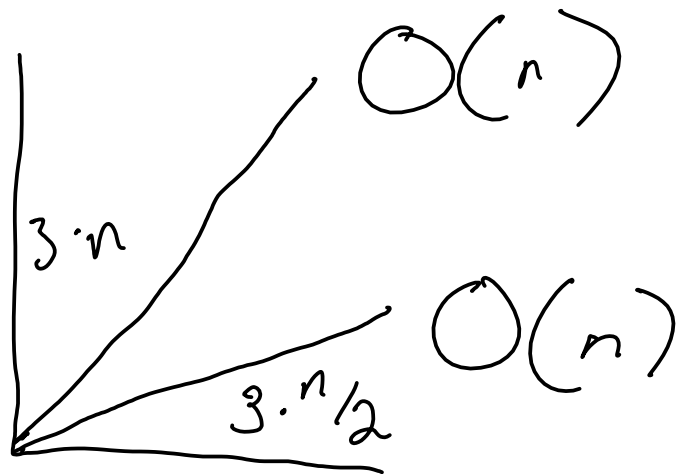
Let  $f(n) = n^2$ . Then  $f(n)$  is  $O(n^3)$ .

$$\forall n \geq 1. 1 \leq n$$

$$\forall n \geq 1. n^2 \leq n^3$$

$$\exists k \geq 1. \forall n \geq k. n^2 \leq n^3 \cdot \textcircled{1}$$

$$\exists c > 0, k \geq 1. \forall n \geq k. n^2 \leq cn^3 \leftarrow$$



Let  $f(n) = 3n$ . Then  $f(n)$  is  $O(n)$ .

$\exists c > 0, k \geq 1. \forall n \geq k. 3n \leq cn$

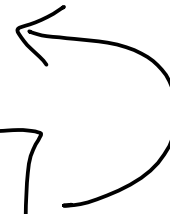
Let  $c = 3, k = 1$ . I + s t s

$$\boxed{\forall n \geq 1. 3n \leq 3n}$$

Let  $f(n) = 2n^2 + n$ . Then  $f(n)$  is  $O(n^2)$ .

$$\exists c > 0, k \geq 1. \forall n \geq k. 2n^2 + n \leq cn^2$$

$$c = 3 \quad k = 1 \quad \forall n \geq 1. 2n^2 + n \leq 3n^2$$

$$\forall n \geq 1. n^2 \geq n \quad \left[ \begin{array}{l} \forall n \geq 1. 2n^2 \leq 2n^2 \\ \forall n \geq 1. n \leq n^2 \end{array} \right]$$


$$\begin{array}{l} a \leq c \\ b \leq d \end{array} \implies a + b \leq c + d$$