

CS46 practice problems 9

These practice problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions**. The purpose of these problems is to get more comfortable with reasoning and writing about Turing machines, decidability, and recognizability.

1. **Infinite languages.** Last week we saw that the following language was decidable:

$$INFINITE_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ is an infinite language}\}$$

(See solved problem 4.10 in the book for a clever way of making this argument.)

- (a) Show that $INFINITE_{CFG}$ is decidable¹, where:

$$INFINITE_{CFG} = \{\langle G \rangle \mid G \text{ is a context-free grammar and } L(G) \text{ is an infinite language}\}$$

- (b) Show that $INFINITE_{TM}$ is not decidable, where:

$$INFINITE_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) \text{ is an infinite language}\}$$

2. **Equal language checking for grammars.**

$$EQ_{CFG} = \{\langle G, H \rangle \mid G \text{ and } H \text{ are context-free grammars and } L(G) = L(H)\}$$

- (a) Show that EQ_{CFG} is co-Turing-recognizable.²

- (b) Show that EQ_{CFG} is undecidable.³

(Note: this is why Automata Tutor for grammars ran thousands of test strings, instead of giving a definite answer!)

¹Hint 1: The clever solution for $INFINITE_{DFA}$ was linked to the pumping lemma for DFAs. How can you use the pumping lemma for context-free languages in a similar way?

Hint 2: The intersection of a context-free language with a regular language is context-free.

²Hint: Use nondeterminism.

³Hint: Theorem 5.13 shows ALL_{CFG} is undecidable; you can use this result without proof.