Name: YOUR NAME HERE

CS46 Homework 12

This homework is due at 11:59pm on Thursday, April 30. This is a 10-point homework.

You may work with one partner on this lab. Your write-up is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking the instructor), then you **must** cite these in your post-lab survey. Please refer to the course webpage or ask me any questions you have about this policy.

1. **Is (almost) everything NP-complete?**

   Show that if P = NP, then every language $A \in \text{P}$ is NP-complete except $A = \emptyset$ and $A = \Sigma^*$.

2. **Please use other door.**

   Give a polynomial time reduction from ThreeColoring to 3-Sat . You may want to use the variables V_{ij} to indicate that vertex $_i$ has color j, $1 \leq j \leq 3$. If we know 3-Sat is NP-complete and ThreeColoring is in NP, what does this reduction tell us?

3. **Vertex cover and independent set, again.**

   Recall that a **vertex cover** in a graph G is a subset of vertices where every edge of G has at least one endpoint in the subset.

   $$\text{VertexCover} = \{\langle G, k\rangle \mid G \text{ has a } k\text{-node vertex cover}\}$$

   Theorem 7.44 says that VertexCover is NP-complete .

   An **independent set** in a graph G is a subset of vertices with no edges between them.

   $$\text{IndependentSet} = \{\langle G, k\rangle \mid G \text{ contains an independent set of } k \text{ vertices}\}$$

   We will show that IndependentSet is NP-complete .

   (a) Prove that IndependentSet $\in$ NP.

   (b) Prove that IndependentSet is NP-hard .
   (Hint: reduce from VertexCover . This is *not* the same direction you did in lab, but you might be able to use the same idea as the core of your reduction.)