CS46, Swarthmore College, Spring 2018
Lab 8 (due Wednesday 11 April)
Name: YOUR NAME(S) HERE

1. Show if $f_1(n)$ is $O(n^{k_1})$ and $f_2(n)$ is $O(n^{k_2})$ where $k_1 \geq k_2$ then $f_1(n) + f_2(n)$ is $O(n^{k_1})$.

2. The familiar linear search algorithm takes as input a list of values $[v_1, v_2, \ldots v_n]$ and a search term x and decides if the $x = v_i$ for some $1 \leq i \leq n$. In the case that the list is empty and $n = 0$, the algorithm can immediately reject. You should recall that linear search has time complexity $f(n) = O(n)$. What is wrong with the following "proof" that linear search has time complexity $f(n) = O(1)$?

   Proof. We will show that linear search has time complexity $O(1)$ by induction on the size of the number of input values n. For the base case, $n = 0$, there is nothing to check and the algorithm clearly runs in $O(1)$ steps. By the inductive hypothesis, assume that linear search runs in $O(1)$ steps for lists up to some size $k < n$. It clearly holds for $k = 0$. For the inductive step, consider the case of a list containing n elements. By the inductive hypothesis, processing the first $n-1$ elements can be done in $O(1)$ time. If x is in the first $n-1$ elements, we accept the input and no further processing is needed. If x is not in the first $n-1$ elements, we spend $O(1)$ time to processes the last element and accept if $x = v_n$ and reject otherwise. The total time spent is $O(1) + O(1) = O(1)$ by the result in previous question. Therefore linear search can be done in $O(1)$ time. □

3. Prove that the complexity class P is closed under concatenation.

4. Prove that the complexity class P is closed under Kleene star. Hint: you may want to consider a technique similar to how we built a table in lab 6 for determining if a grammar in Chomsky Normal Form could generate a string w. You probably showed decidable or recognizable languages were closed under Kleene star using non-determinism. Languages in P can only use deterministic Turing machines that run in polynomial time.

5. Given a graph $G = (V, E)$ with a set of vertices V and edges E, we say that G has 3-clique if there exist three vertices $a, b, c \in V$ such that there is an edge $e \in E$ for every pair of unique vertices in $\{a, b, c\}$. In other words, G contains a triangle. Show that the language $L = \{\langle G = (V, E)\rangle | G \text{ contains a 3-clique } \}$ is in P.