

CS46, Swarthmore College, Spring 2018

Lab 5 (due Wednesday 28 February)

Name: YOUR NAME(S) HERE

1. If  $A$  and  $B$  are languages, define  $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$ .
  - (a) Show that if  $A$  and  $B$  are regular languages, then  $A \diamond B$  is a CFL. You will probably want to describe a PDA instead of a grammar and use non-determinism. You do not need to give a complete definition of your PDA, but instead describe at a high level how it works.
  - (b) If  $A$  and  $B$  are regular languages, is  $A \diamond B$  regular? If so, give a proof. If not, give an example of two regular languages  $A$  and  $B$  for which  $A \diamond B$  is not regular, and prove your claim.
2. The book shows that  $L = \{ww \mid w \in \Sigma^*\}$  is not context free, for  $\Sigma = \{a, b\}$ . Design a grammar to show that the complement of  $L$ ,  $\bar{L}$  is context free. In addition to providing your grammar  $G$ , you should provide a proof or proof sketch that your grammar generates  $\bar{L}$ . Be sure to sketch both directions: if  $G$  generates  $w$  then  $w \in \bar{L}$  and if  $w \in \bar{L}$  then  $G$  generates  $w$ . This construction shows that context free languages are not closed under complement.
3. (Sipser 2.6d): Give a context-free grammar that generates the following language:  
 $\{x_1 \# x_2 \# \cdots \# x_k \mid k \geq 1, \text{ each } x_i \in \{a, b\}^* \text{ and for some } i \text{ and } j, x_i = x_j^R\}$ . Note that (i)  $i$  and  $j$  are not necessarily distinct, (ii) any palindrome  $x_i$  satisfies  $x_i = x_i^R$ , (iii) and that  $|x_i|$  can be zero for any  $i$ .
4. (Sipser 2.30d): Use the pumping lemma for context free languages to show the following language is not context free:  $\{x_1 \# x_2 \# \cdots \# x_k \mid k \geq 2, \text{ each } x_i \in \{a, b\}^* \text{ and } x_i = x_j \text{ for some } i \neq j\}$ .