CS46, Swarthmore College, Spring 2014
Homework 8 – due 24 April
Your Name(s) Here

1. The complexity class *coNP* consists of languages whose complement is in NP. It is currently an open question if $coNP = NP$. Show if $NP \neq coNP$ then $P \neq NP$.

2. Show that if $P = NP$ then any language $A \in P$ where $A \neq \emptyset$ and $A \neq \Sigma^*$ is NP-complete.

3. Give a reduction from *3-COLOR* to *3-SAT*. If we know *3-SAT* is NP-complete, what do we know about *3-COLOR* based on this reduction?

4. (L&P 6.3.3) Consider a Boolean formula in 2-CNF. Any clause $(x \vee y)$ can be thought of as two implications $\overline{x} \implies y$ and $\overline{y} \implies x$. The clause (x) can be thought of as $\overline{x} \implies x$. If we then consider $x \implies y$ as a directed edge from a vertex x to a vertex y, we can construct an implication graph from any 2-CNF formula. Show that a 2-CNF formula is unsatisfiable if and only if there is a variable x such that there is a path in the constructed graph from x to $\overline{x}$ *and* a path from $\overline{x}$ to x. Design an algorithm based on this observation to show *2-SAT* $\in P$