CS46, Swarthmore College, Spring 2014
Homework 4 (due Thursday 20 February)
Your Name(s) Here

No programming this week, only a written portion.

1. Sipser 1.48: Let $\Sigma = \{0, 1\}$ and let $L = \{w | w$ contains an equal number of occurrences of the substrings 01 and 10 $\}$ Show that $L$ is regular. Note $101 \in L$ but $1010 \notin L$.

2. Let $\Sigma = \{a, b\}$.

   (a) Let $A = \{a^k u a^k | k \geq 1$ and $u \in \Sigma^*$. Show that $A$ is regular.

   (b) Let $B = \{a^k b u a^k | k \geq 1$ and $u \in \Sigma^*$. Show that $B$ is not regular.

3. Binary addition is regular, but multiplication is not. Let our alphabet $\Sigma$ be the set of all size 3 binary vectors:

$$\Sigma = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

   A correct multiplication of two binary numbers can be represented by a string in $\Sigma^*$. For example:

$$\begin{array}{rcccccc} & 0 & 0 & 1 & 0 & 1 & 0 \\ \times & 0 & 0 & 0 & 1 & 0 & 1 \\ \hline & 1 & 1 & 0 & 0 & 1 & 0 \end{array}$$

   would be represented by the following string of six symbols from $\Sigma$:

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

   Let the language $L$ be the set of all strings in $\Sigma^*$ representing correct binary multiplications. Use the pumping lemma to show that $L$ is not regular.

4. Construct context-free grammars that generate each of these languages:

   (a) $\{wcw^R : w \in \{a, b\}^*\}$

   (b) $\{ww^R : w \in \{a, b\}^*\}$

   (c) $\{w \in \{a, b\}^* : w = w^R\}$

5. Recall the definition of regular expressions given on Sipser page 64. Give a formal description of a context-free grammar that generates the language

$$L = \{R : R \text{ is a regular expression for the alphabet } \{a, b\}\}$$