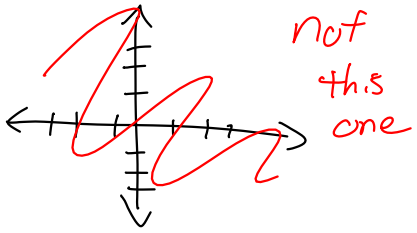
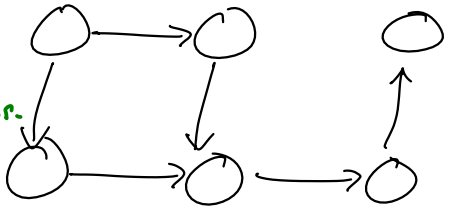
Graphs

ADT — abstract data type

- describes available operations of a data container
- does not describe how

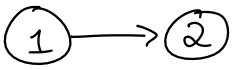


Graphs are good at describing relationships.

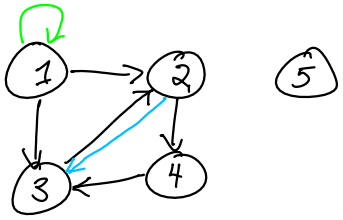


A graph is a pair of sets $\langle V, E \rangle$.

- V is a set of vertices (vertex / node)
- E is a set of edges, which are pairs* of soure vertex and target (destination) vertex



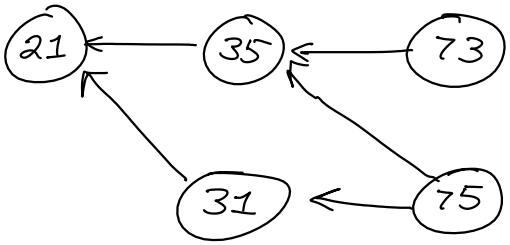
$V = \{1, 2\}$ $E = \{\langle 1,2 \rangle\}$



$V = \{1, 2, 3, 4, 5\}$

$E = \{\langle 1,2 \rangle, \langle 3,2 \rangle, \langle 1,3 \rangle, \langle 4,3 \rangle, \langle 2,4 \rangle\}$

$\langle 2,3 \rangle$ $\langle 1,1 \rangle$



V = set of CS courses

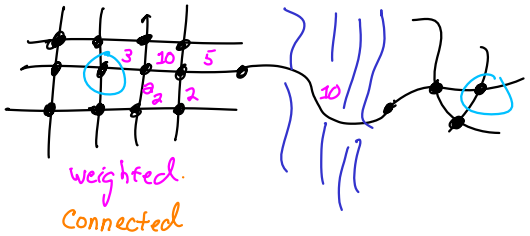
E = set of prerequisites

edge = requires

Graph Examples

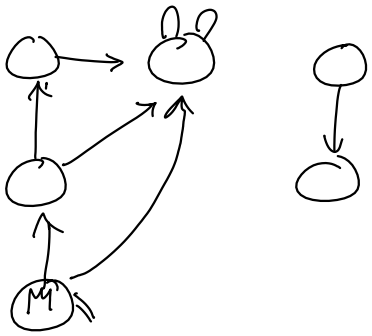
Maps

undirected



weighted
connected

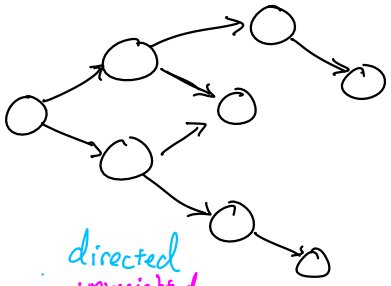
Food Chain



V = animal
E = eats

directed
unweighted
disconnected

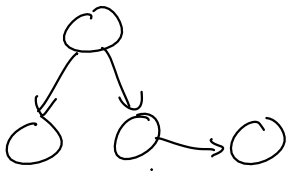
Decision Stories



V = decision point
E = choice ↦ state

directed
unweighted

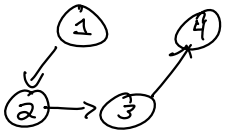
Evolution



V = species
E = evolves to

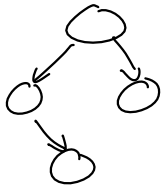
directed
(?)

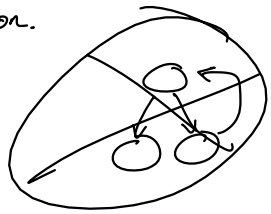
[<1,2>, <2,3>, <3,4>]



An undirected graph is one in which, for every $\langle V_1, V_2 \rangle \in E$ (in E), $\langle V_2, V_1 \rangle \in E$ (in E).
All other graphs are directed.

A weighted graph is one in which each edge carries a weight describing an aspect of the relationship.
A unweighted graph does not carry weights. (A "label" carries relationship info. Not necessarily comparable, numeric, etc.)
A path is a list of edges in a graph such that each edge's destination is the next edge's source.
A graph is connected if, for any two vertices, there is a path between them.
A graph is weakly connected if it would be connected if it were undirected.
The in-degree of a vertex is the number of edges with this vertex as destination.
A tree is a kind of graph.





Trees
- are weakly connected
- have in-degree of at most 1.
- acyclic (no path starting and ending at same vertex)

A simple graph is a graph which has
- No self-loops (no edges of form $\langle V_1, V_1 \rangle$)
- Has at most one edge from a given vertex to another