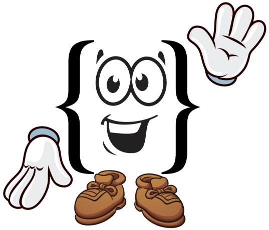
Demand-Driven Relative Store Fragments for Singleton Abstraction

Little Store's Big Journey



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The Johns Hopkins University[1]

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September 1st, 2017

Some Program Analyses

	push forward	reverse lookup
first order	classic abs. interp. data flow analysis	CFL-reachability reverse data flow analysis
higher order	kCFA CFA2 PDCFA ΓCFA	

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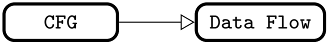
Some Program Analyses

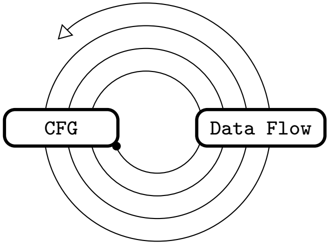
First Order

`for (int i=0;i<n;i++)`



Higher Order

`fold (λa e -> ...)`



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Some Program Analyses

	push forward	reverse lookup
first order	classic abs. interp. data flow analysis	CFL-reachability reverse data flow analysis
higher order	kCFA CFA2 PDCFA ΓCFA	DDPA **DRSF** POLYFLOW$_{\mathrm{CFL}}$ (weak non-locals)

Demand-Driven Higher-Order Program Analyses

	DDPA	**DRSF**
Context-sensitive		
Flow-sensitive		
Path-sensitive		
Must-alias		
Non-local variables		

Demand-Driven Higher-Order Program Analyses

	DDPA	DRSF
Context-sensitive	✓	
Flow-sensitive	✓	
Path-sensitive	∼	
Must-alias	∼	
Non-local variables	✓	

Demand-Driven Higher-Order Program Analyses

	DDPA	DRSF
Context-sensitive	✓	✓
Flow-sensitive	✓	✓
Path-sensitive	∼	✓
Must-alias	∼	✓
Non-local variables	✓	✓

Demand-Driven Higher-Order Program Analyses

	DDPA	DRSF
Context-sensitive	✓ Contours	✓
Flow-sensitive	✓	✓
Path-sensitive	∼	✓
Must-alias	∼	✓
Non-local variables	✓	✓

Demand-Driven Higher-Order Program Analyses

	DDPA	**DRSF**
Context-sensitive	✓ Contours	✓
Flow-sensitive	✓ Natural	✓
Path-sensitive	∼	✓
Must-alias	∼	✓
Non-local variables	✓	✓

Demand-Driven Higher-Order Program Analyses

	DDPA		DRSF
Context-sensitive	✓	Contours	✓
Flow-sensitive	✓	Natural	✓
Path-sensitive	∼	Filters	✓
Must-alias	∼		✓
Non-local variables	✓		✓

Demand-Driven Higher-Order Program Analyses

	DDPA	DRSF
Context-sensitive	✓ Contours	✓
Flow-sensitive	✓ Natural	✓
Path-sensitive	∼ Filters	✓
Must-alias	∼ A Mess	✓
Non-local variables	✓	✓

Demand-Driven Higher-Order Program Analyses

	DDPA	**DRSF**
Context-sensitive	✓ Contours	✓
Flow-sensitive	✓ Natural	✓
Path-sensitive	∼ Filters	✓
Must-alias	∼ A Mess	✓
Non-local variables	✓ Lookup	✓

Demand-Driven Higher-Order Program Analyses

	DDPA	DRSF
Context-sensitive	✓ Contours	✓ Little Stores
Flow-sensitive	✓ Natural	✓ Little Stores
Path-sensitive	∼ Filters	✓ Little Stores
Must-alias	∼ A Mess	✓ Little Stores
Non-local variables	✓ Lookup	✓ Lookup

DDPA by Example

DDPA by Example

```
let f = fun p ->
    let x = p in
    fun y -> x + y
in
let g = f 4 in
let v = 1 in
let z = g v in z
```

DDPA by Example

Initial CFG

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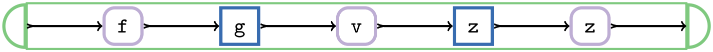
```



DDPA by Example

Expand function call f 4

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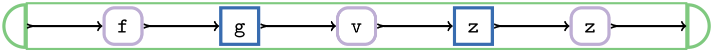

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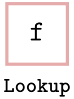

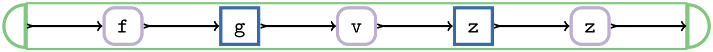

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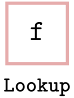

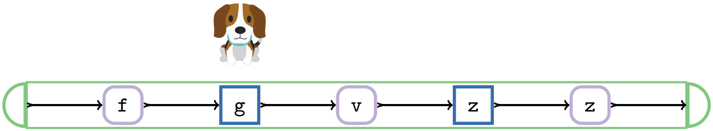

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Expand function call `f 4`

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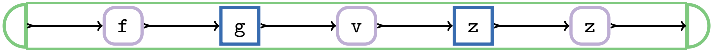
Lookup

"Look Pup"



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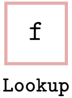

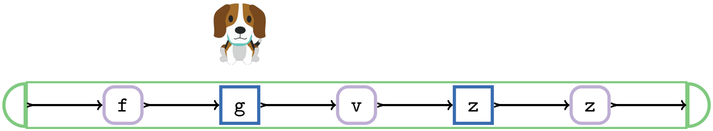

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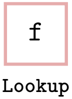

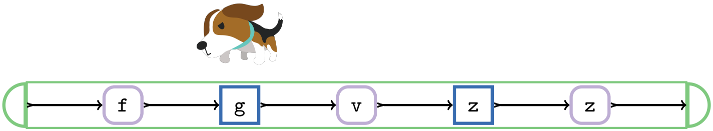

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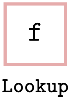

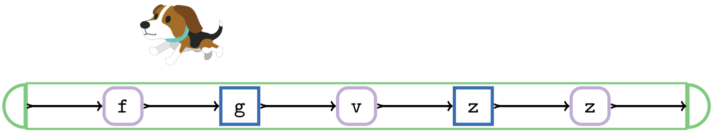

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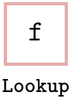

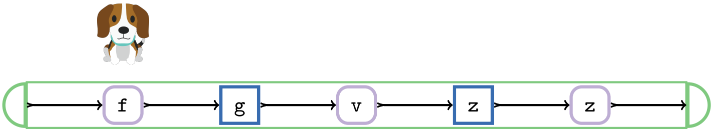

DDPA by Example

Wire in function call `f 4`

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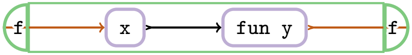

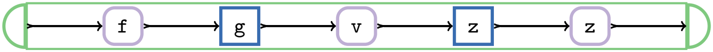

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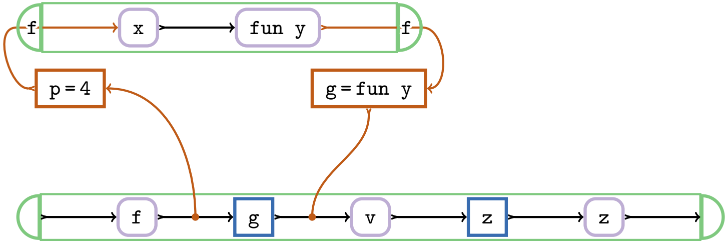

DDPA by Example

Expand function call g v

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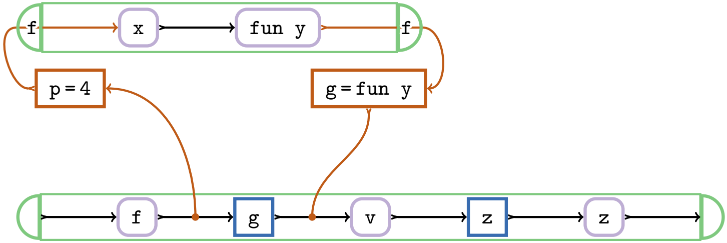

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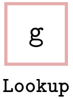

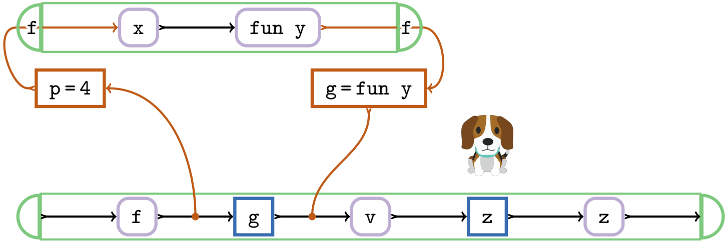

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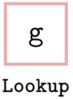

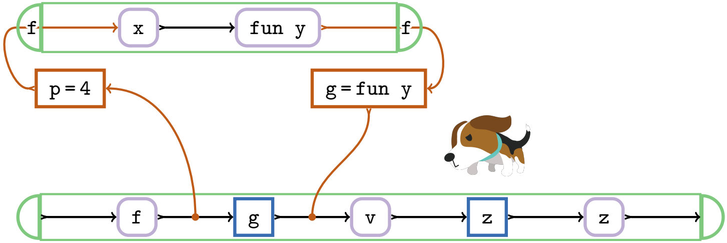

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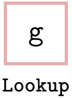

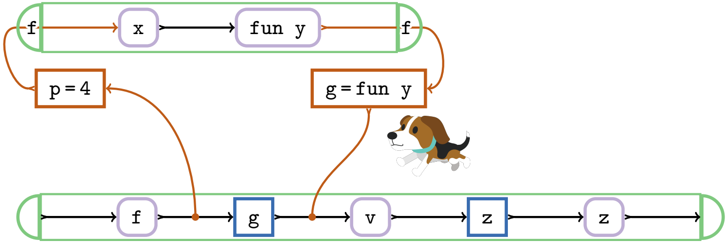

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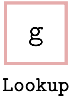

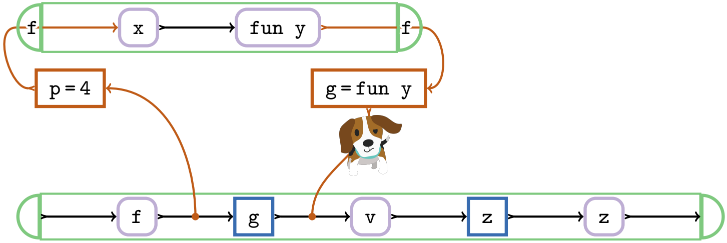

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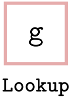

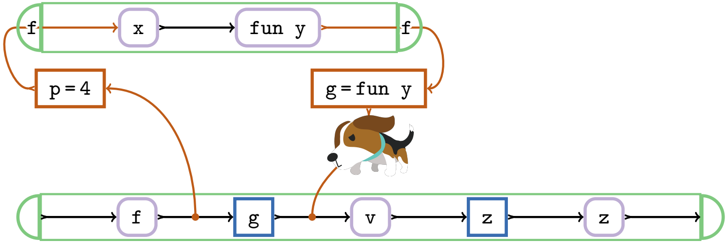

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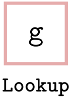

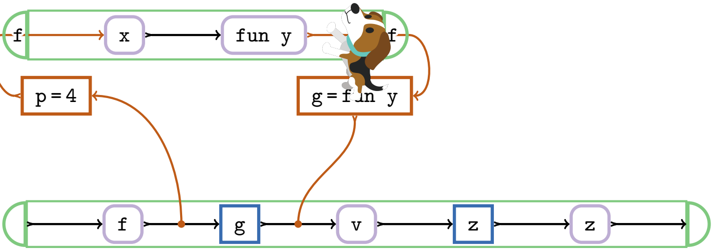

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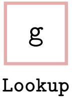

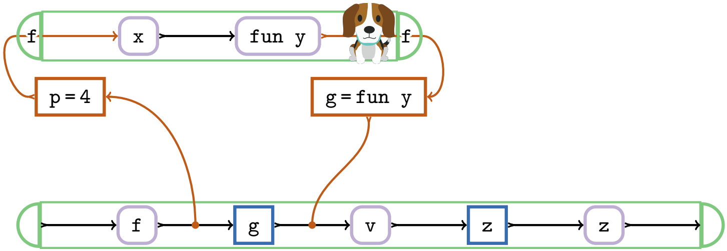

DDPA by Example

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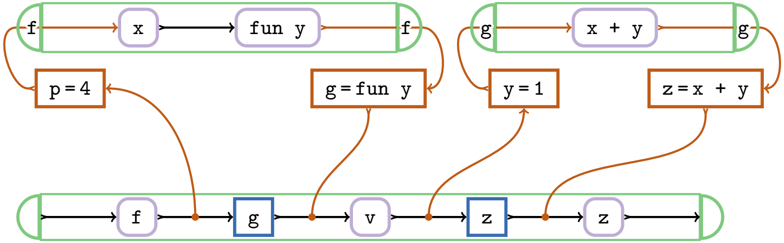

DDPA by Example

Parameter lookup: y

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let f = fun p ->
    let x = p in
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```

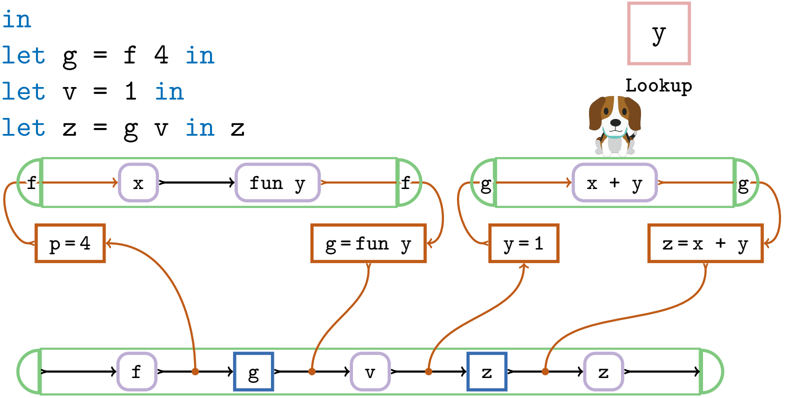

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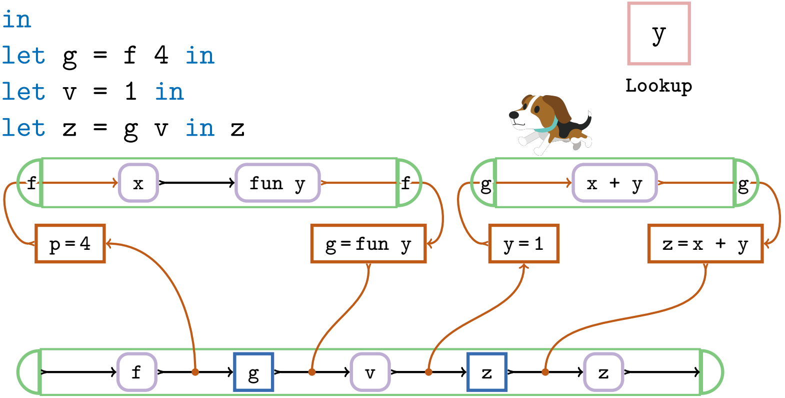

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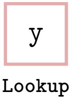

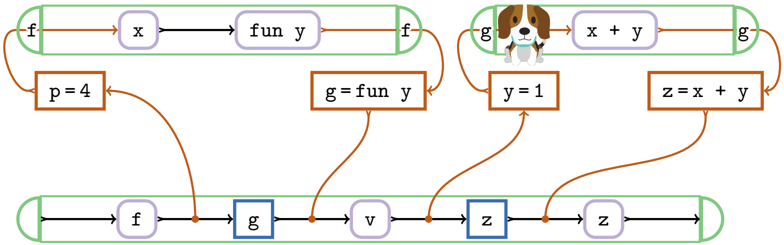

DDPA by Example

Non-local lookup: x

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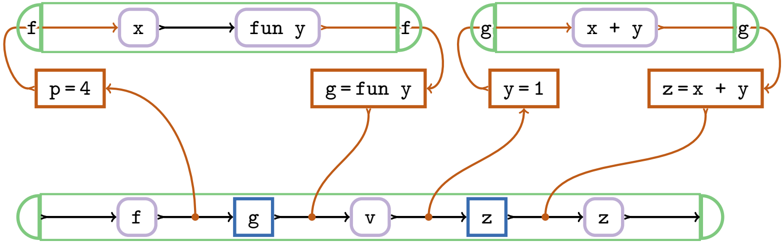

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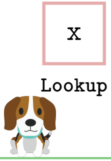

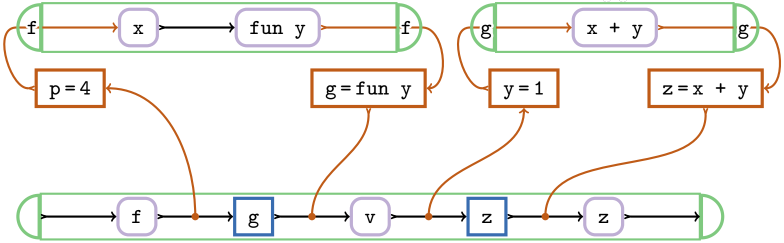

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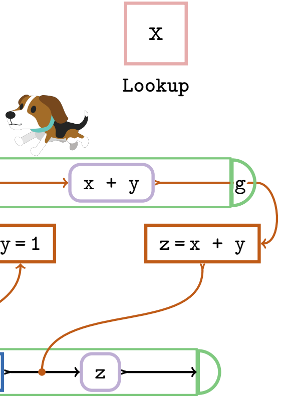

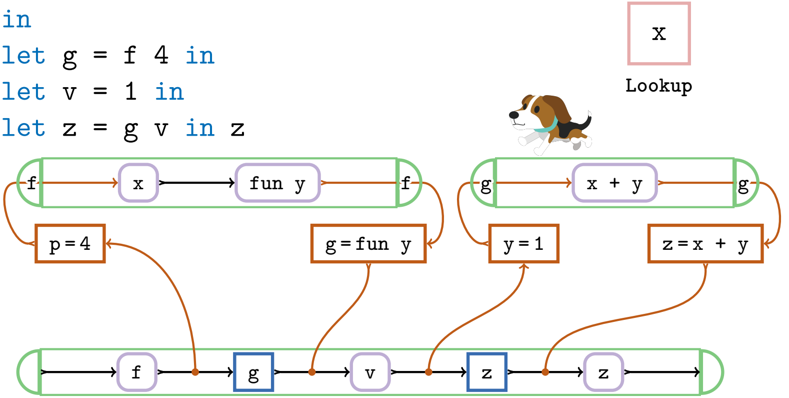

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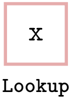

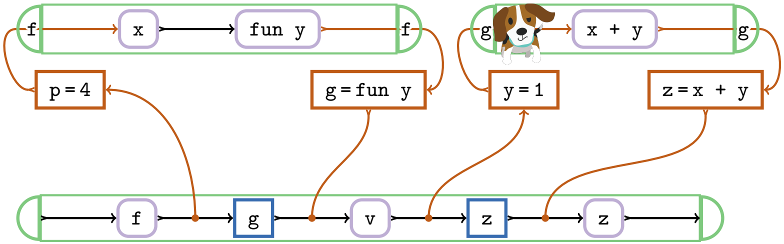

DDPA by Example

Non-local lookup: `x`

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let f = fun p ->
    let x = p in
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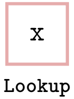
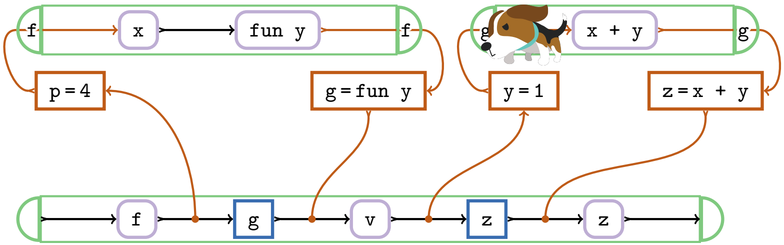
```





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```
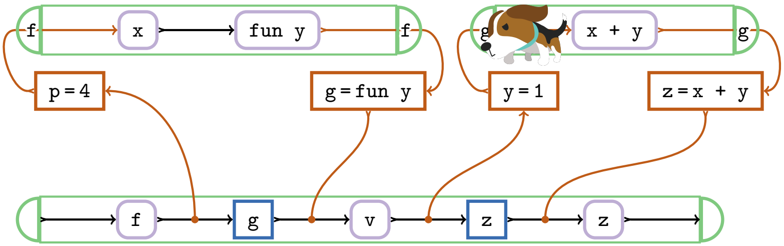

g
x

Lookup Stack



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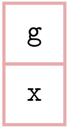

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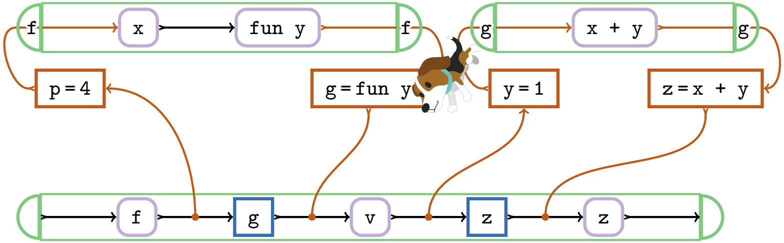

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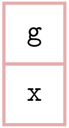

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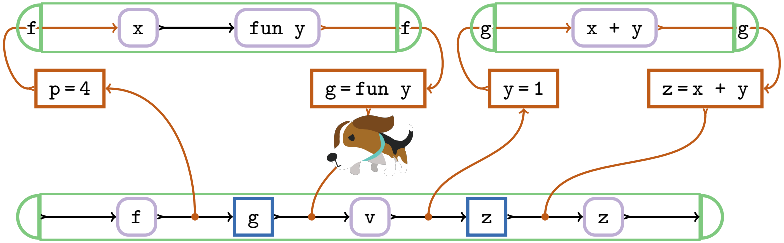

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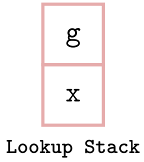

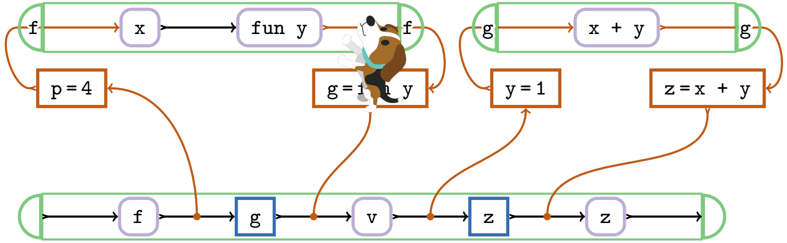

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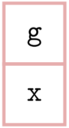

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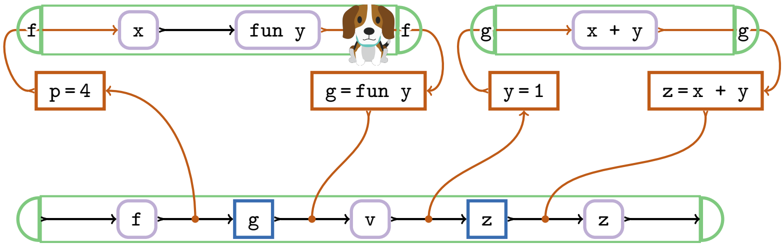

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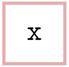

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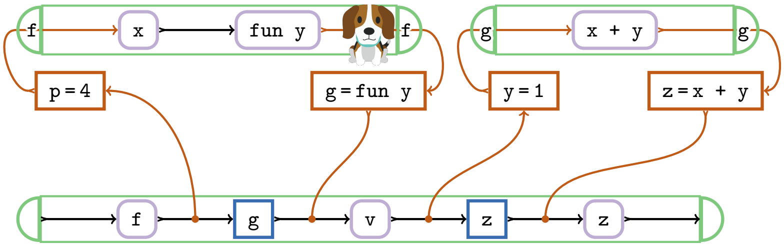

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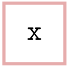

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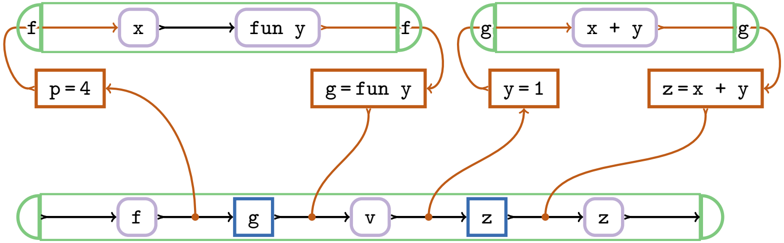

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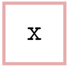

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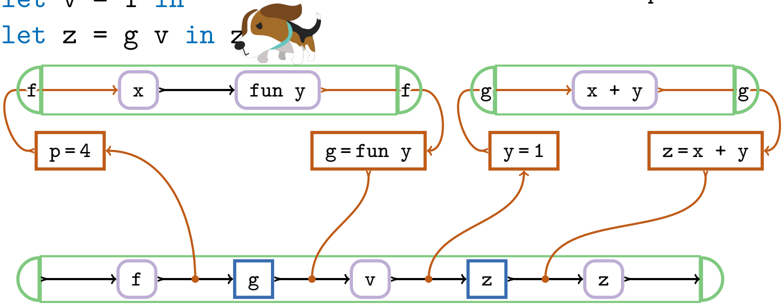

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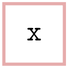

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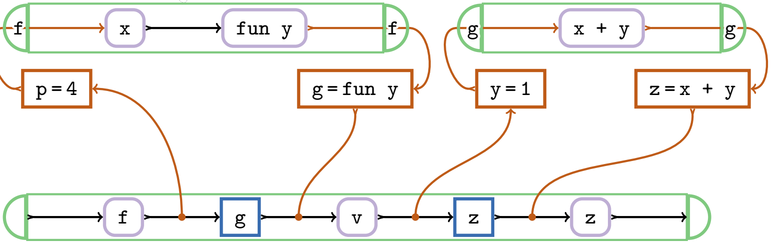

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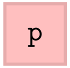
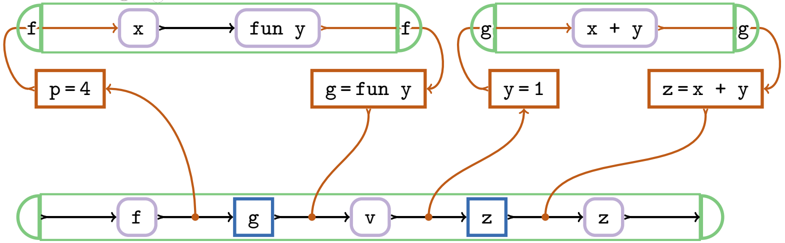


Lookup Stack



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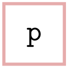
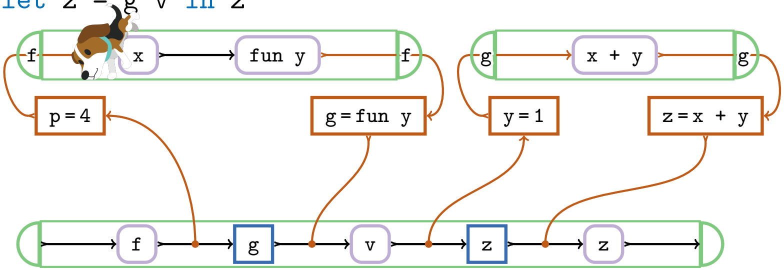


Lookup Stack



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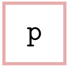

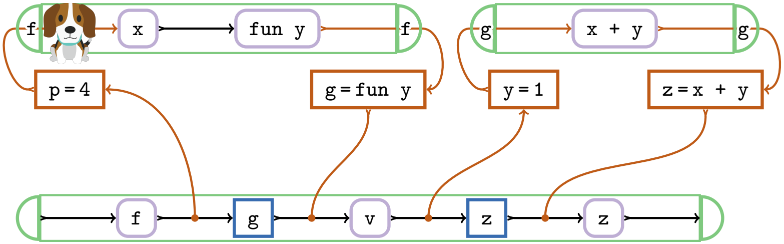

DDPA

- Value lookup on demand: no explicit store!

DDPA

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- Lookup stack: intermediate lookups

DDPA

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  - Record projections
  - Binary operators
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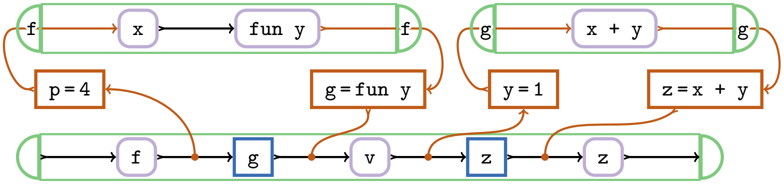
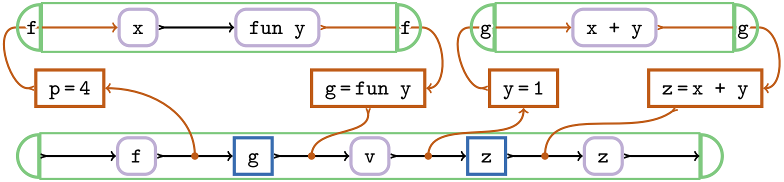
Connection to forward analyses?

DDPA and Abstract Stores



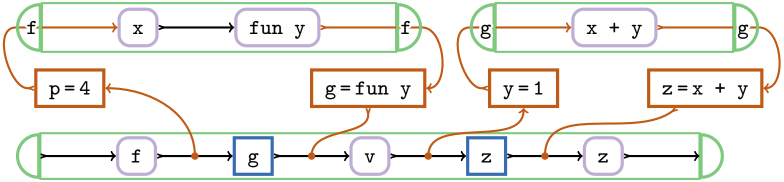
DDPA and Abstract Stores



$\{\}$

DDPA and Abstract Stores

$\{\texttt{p} \mapsto 4\}$



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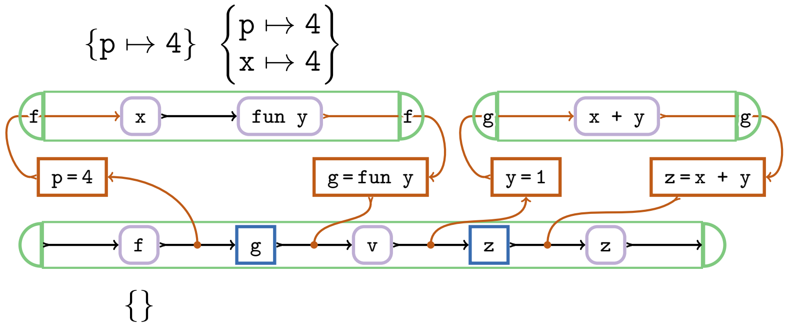
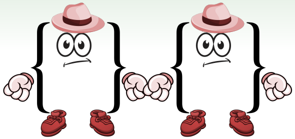
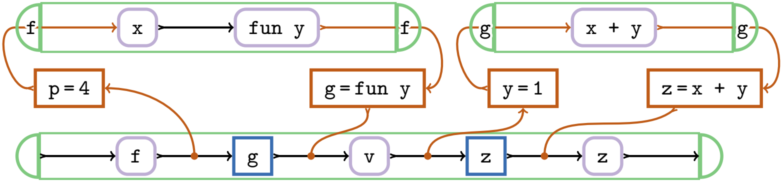
DDPA and Abstract Stores



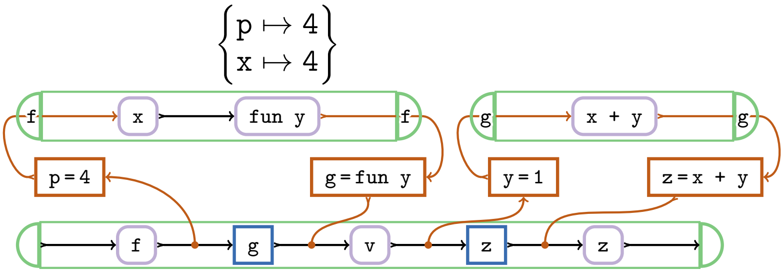
$\{p \mapsto 4\}$
$\left\{\begin{matrix} p \mapsto 4 \\ x \mapsto 4 \end{matrix}\right\}$
f
x
fun y
f
g
x + y
g
p = 4
g = fun y
y = 1
z = x + y
f
g
v
z
z
$\{\}$





- "Big stores": complete sets of bindings

DDPA and Abstract Stores



$\left\{\begin{matrix} p \mapsto 4 \\ x \mapsto 4 \end{matrix}\right\}$
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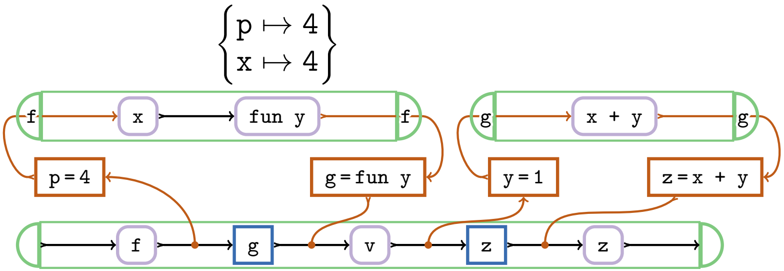
z
z

- "Big stores": complete sets of bindings
- DDPA: reconstruct big stores with lookups

DDPA and Abstract Stores



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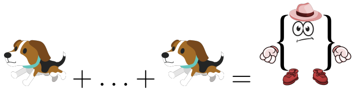
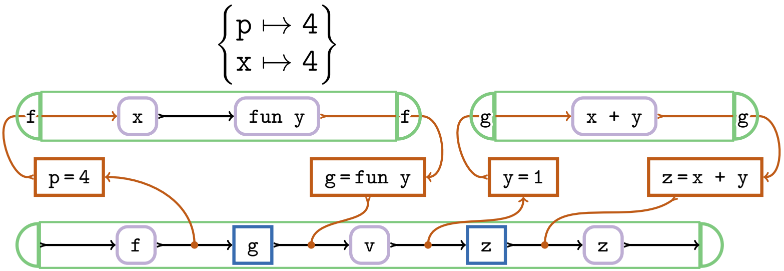
- 

DDPA and Abstract Stores



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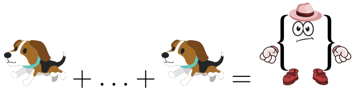
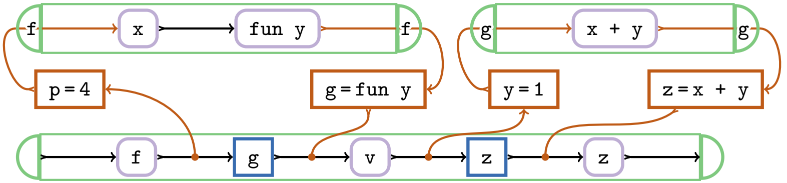
  - 
- Lookups from a point are independent

DDPA and Abstract Stores

$$\left\{\begin{matrix} p \mapsto 4 \\ x \mapsto 4 \end{matrix}\right\}$$



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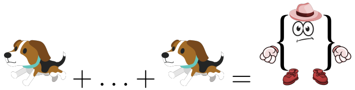
  - 

- Lookups from a point are independent
- Similar to per-point store widening

DDPA and Variable (Mis-)Alignment

```
let f = fun p ->
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DDPA and Variable (Mis-)Alignment

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let b = coin_flip () in
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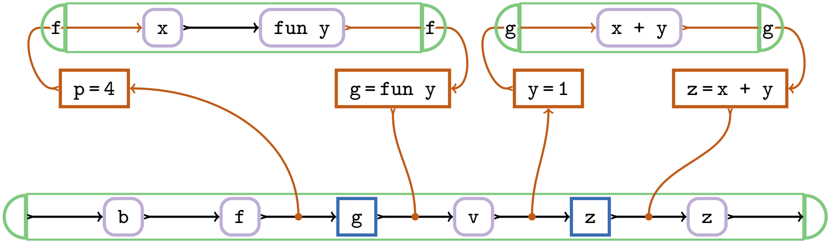

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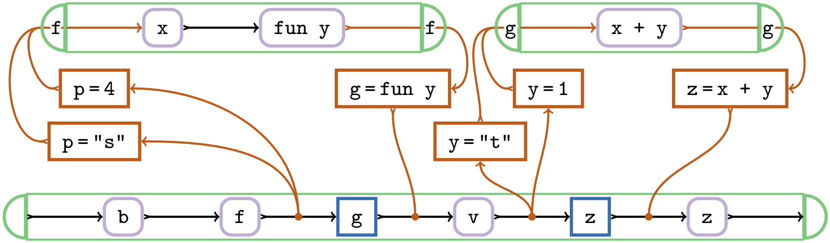

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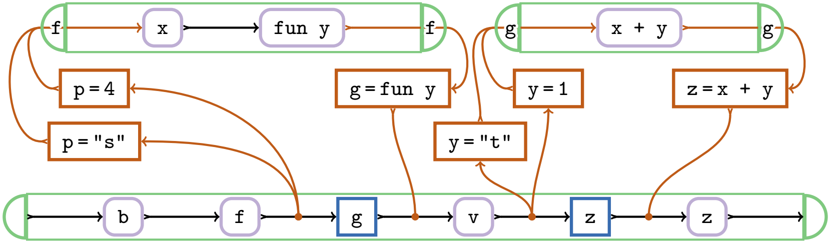
Possible values of x + y?



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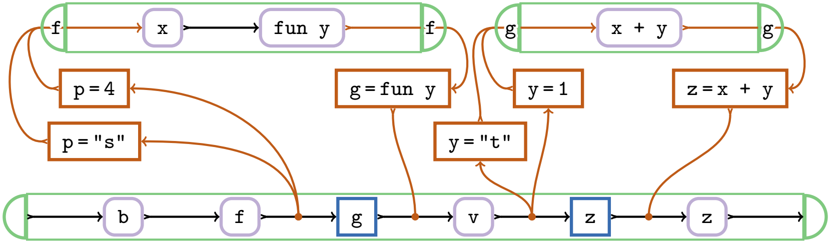
Possible values of `x + y`?

$x \in \{4, \texttt{"s"}\}$



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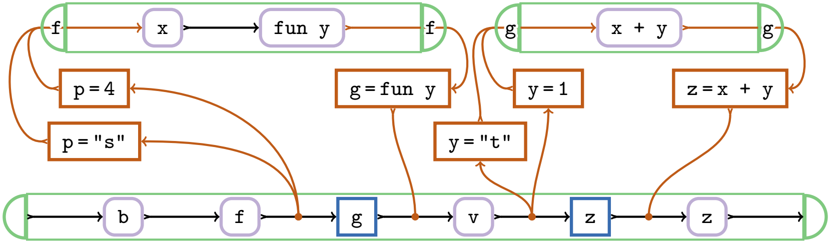
$x \in \{4, "s"\}$

$y \in \{1, "t"\}$



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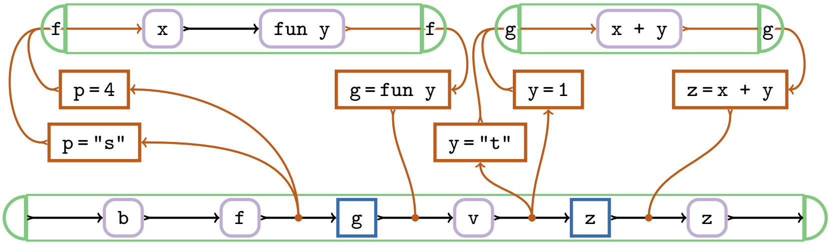
Possible values of `x + y`?

$\texttt{x} \in \{4, \texttt{"s"}\}$

$\texttt{y} \in \{1, \texttt{"t"}\}$

$$\texttt{x + y} = \left\{ \begin{array}{ccc} 4 & + & 1 \\ 4 & + & \texttt{"t"} \\ \texttt{"s"} & + & 1 \\ \texttt{"s"} & + & \texttt{"t"} \end{array} \right\}$$



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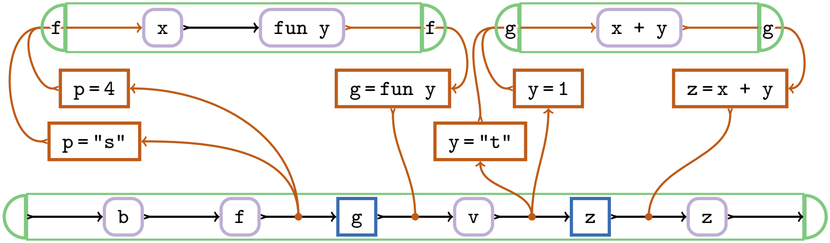
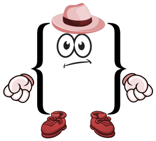
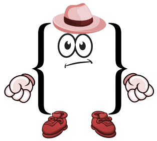


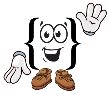
DRSF

DRSF

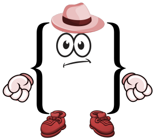
$\bigcup$  $= \sum$

DRSF

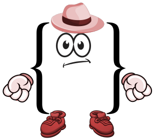
$\bigcup$  $=\sum$



DRSF

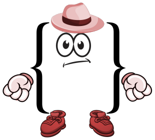
$\bigcup$  $= \sum$ $\neq$

DRSF

$\bigcup$  $= \sum$ $\neq$

$= \{\hat{x}@\Delta \mapsto \hat{v}, \ldots\}$

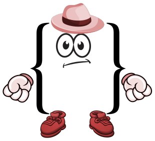
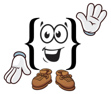
DRSF

$\bigcup$  $= \sum$ $\neq$

$= \{\hat{x} @ \Delta \mapsto \hat{v}, \ldots\}$

$$\Delta = [\delta, \ldots]$$

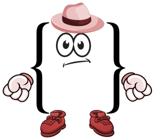
DRSF

$\bigcup$  $= \sum$ $\neq$ 

$= \{\hat{x} @ \Delta \mapsto \hat{v}, \ldots\}$

$\Delta = [\delta, \ldots] \quad \delta ::= ◖\mathtt{x} \mid ◗\mathtt{x}$

DRSF

$\bigcup$  $= \sum$ $\neq$

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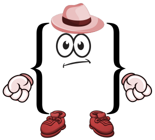
- ΔCFA [POPL 06] (abstract frame strings)
- PDCFA [JFP #24 (2014)] (stack deltas, reachability)

DRSF

$\bigcup$  $= \sum$ $\neq$

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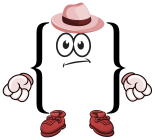
- ΔCFA [POPL 06] (abstract frame strings)
- PDCFA [JFP #24 (2014)] (stack deltas, reachability)
- Little stores are <u>incomplete</u>

DRSF

$\bigcup$  $=\sum$ $\neq$

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- ΔCFA [POPL 06] (abstract frame strings)
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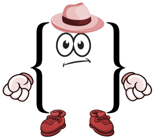
- Little stores are incomplete
- Relative (vs. DDPA's absolute)

DRSF

$\bigcup$  $= \sum$ $\neq$

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- ΔCFA [POPL 06] (abstract frame strings)
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- Little stores are incomplete
- Relative (vs. DDPA's absolute)

Demand-Driven Higher-Order Program Analyses

	DDPA	**DRSF**
Context-sensitive	✓ Contours	✓ Little Stores
Flow-sensitive	✓ Natural	✓ Little Stores
Path-sensitive	∼ Filters	✓ Little Stores
Must-alias	∼ A Mess	✓ Little Stores
Non-local variables	✓ Lookup	✓ Lookup

DRSF and Variable Alignment

```
let b = coin_flip () in
let f = fun p ->
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let g = f (b?4:"s") in
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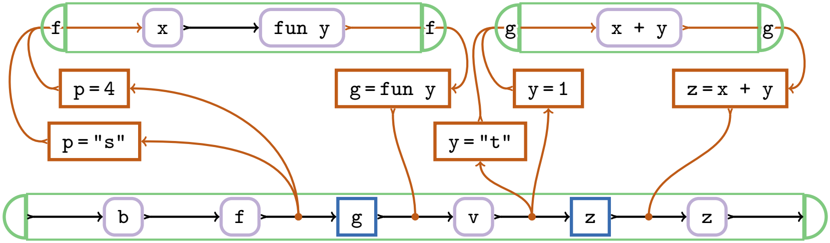

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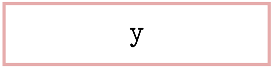
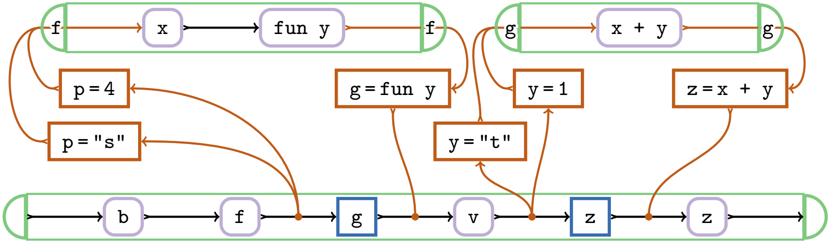


Lookup Stack



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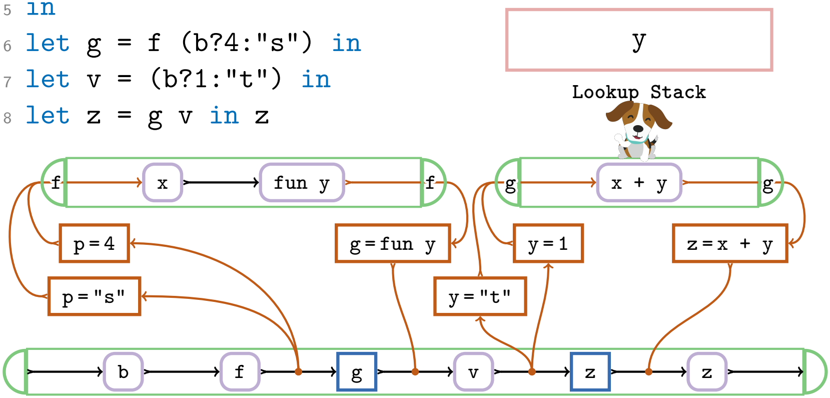

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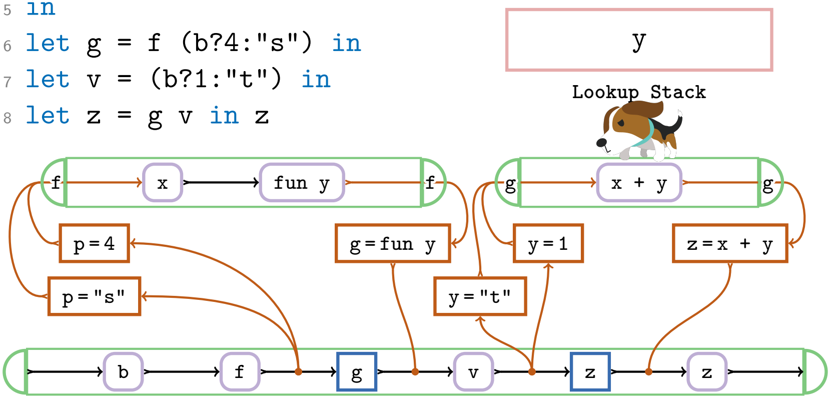

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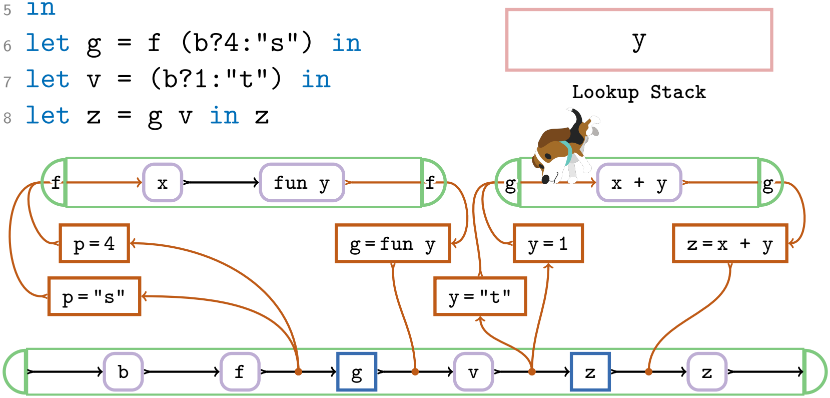

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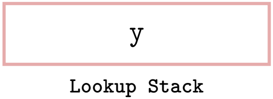

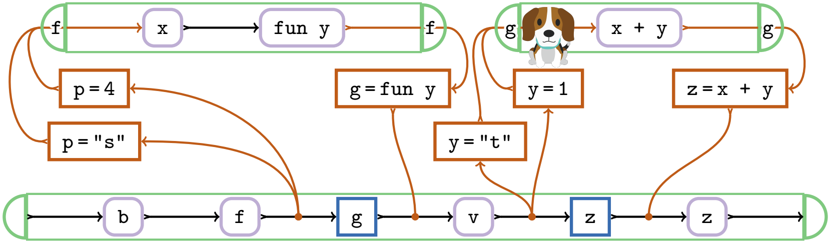

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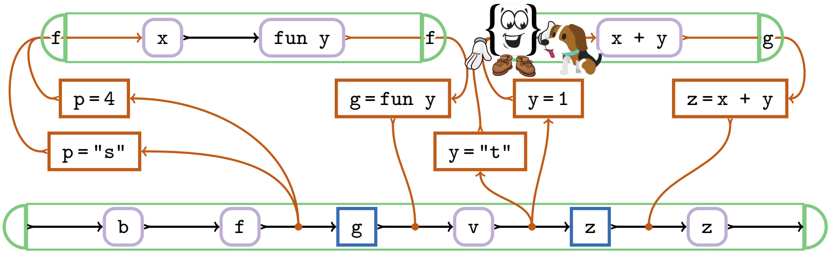
```

$\{y@[] \mapsto 1\}$

Lookup Stack



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```
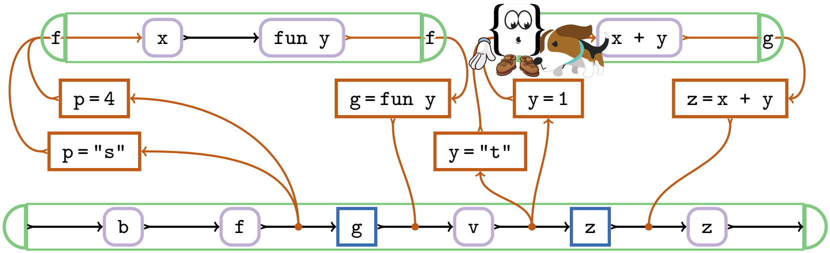

b
$\{y@[] \mapsto 1\}$

Lookup Stack



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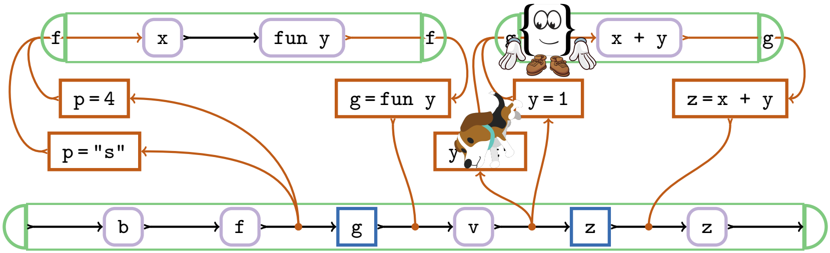
b
$\{y@[] \mapsto 1\}$

Lookup Stack



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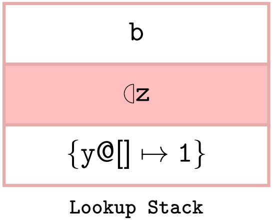

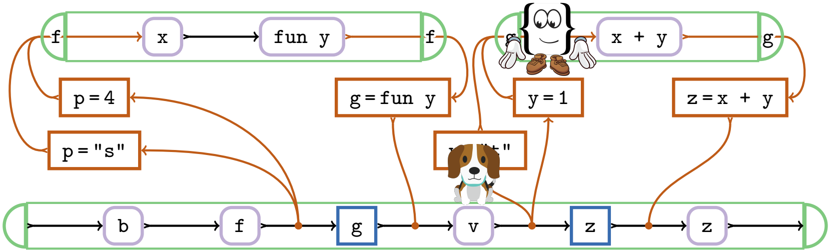

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let g = f (b?4:"s") in
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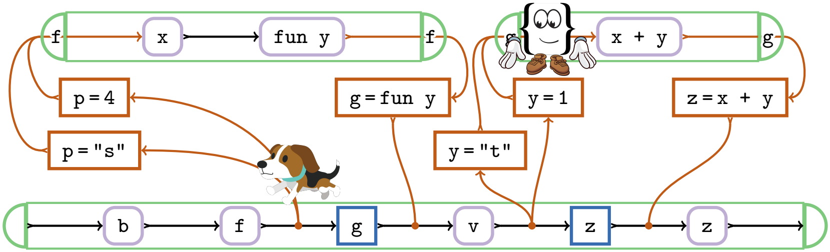
let z = g v in z
```

b
◖z
{y@[] ↦ 1}

Lookup Stack



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```
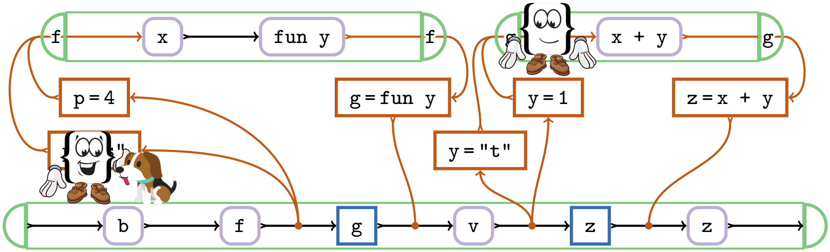

$\{$b@[] $\mapsto$ true$\}$
◖z
$\{$y@[] $\mapsto$ 1$\}$

Lookup Stack



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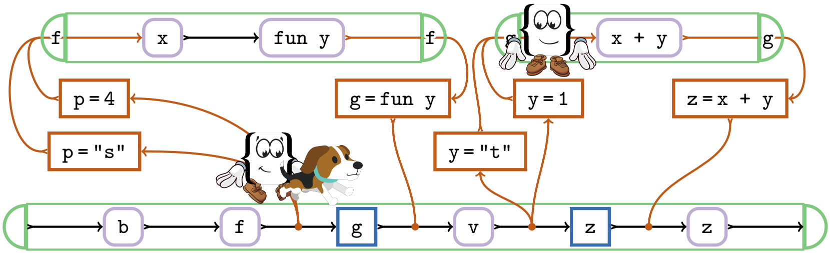
```

{b@[] ↦ true}
◖z
{y@[] ↦ 1}

Lookup Stack



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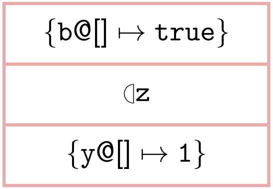
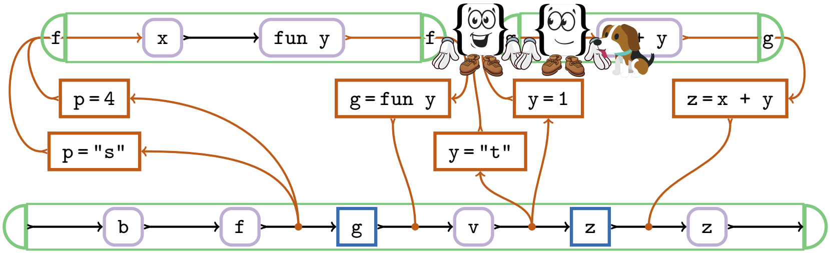


Lookup Stack



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```
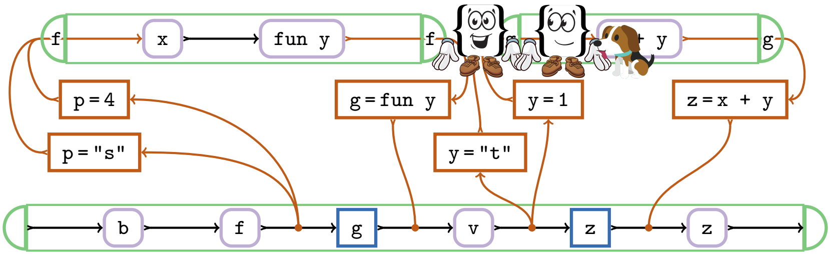

$\{b@[\triangleleft z] \mapsto \texttt{true}\}$

$\{y@[] \mapsto 1\}$

Lookup Stack



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$\{b@[\langle z] \mapsto \texttt{true}\}$
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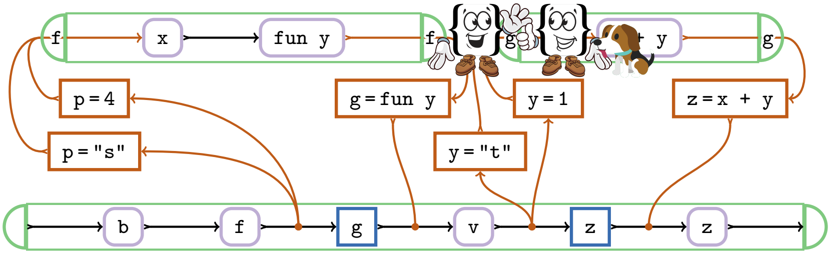
Lookup Stack



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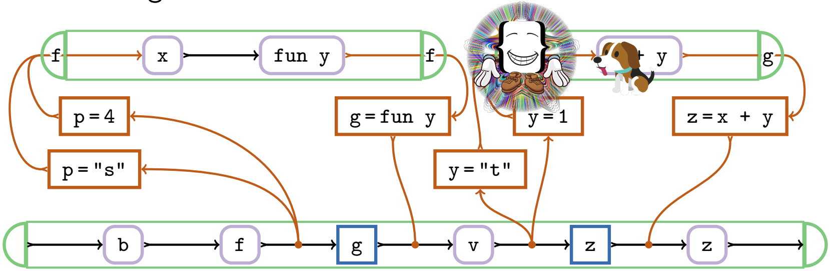
```

$$\left\{ \begin{array}{c} \texttt{y@[]} \mapsto \texttt{1}, \\ \texttt{b@[◖z]} \mapsto \texttt{true} \end{array} \right\}$$

Lookup Stack



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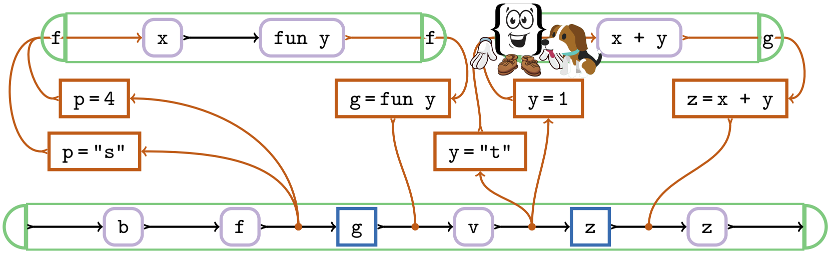
$$\left\{\begin{array}{c} \texttt{y}@[] \mapsto \texttt{1}, \\ \texttt{b}@[\langle\texttt{z}] \mapsto \texttt{true} \end{array}\right\}$$

Lookup Stack



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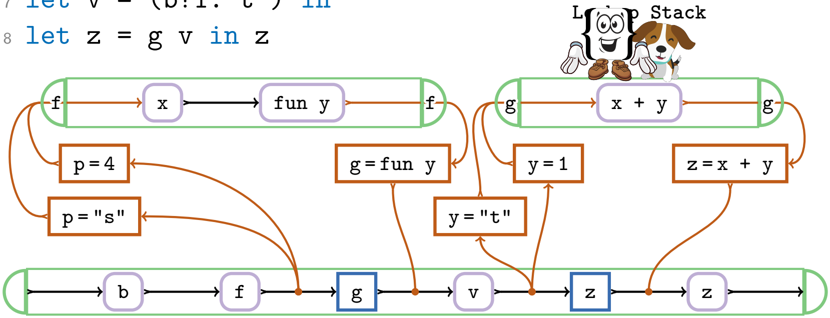
$$\left\{ \begin{array}{c} \mathtt{y}@[] \mapsto 1, \\ \mathtt{b}@[\triangleleft \mathtt{z}] \mapsto \mathtt{true} \end{array} \right\}$$



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```
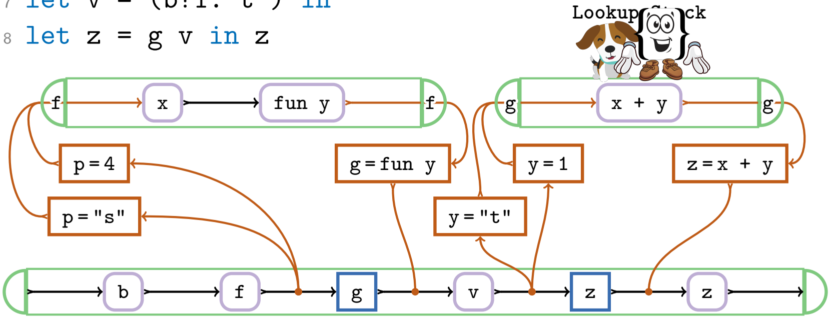

$$\left\{\begin{array}{c} y@[] \mapsto \texttt{"t"}, \\ b@[\lhd z] \mapsto \texttt{false} \end{array}\right\}$$



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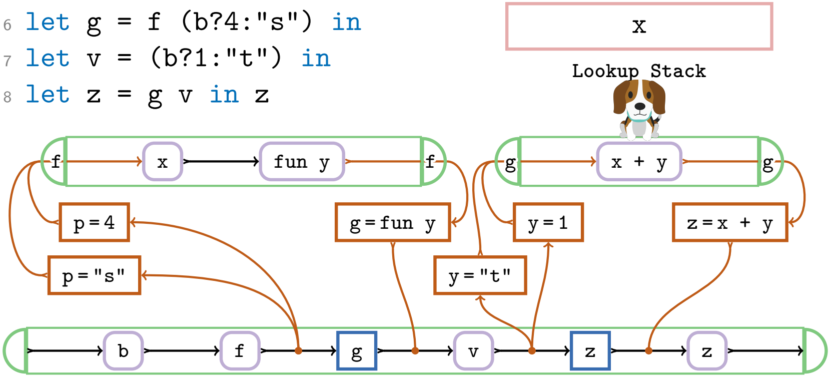

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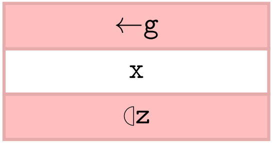
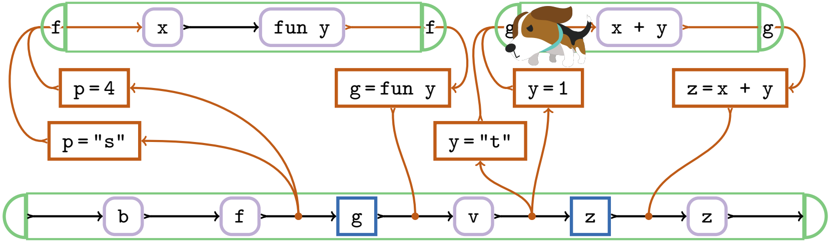


Lookup Stack



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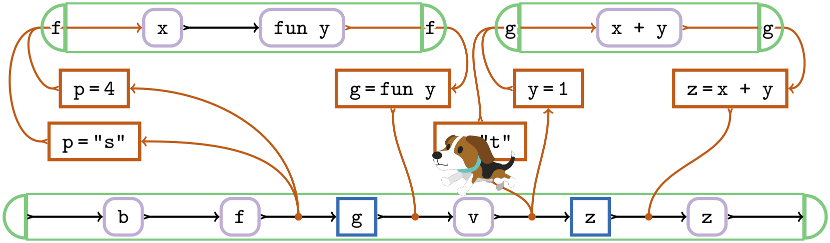
```

←g
x
◖z

Lookup Stack



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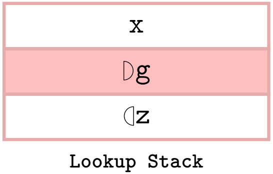

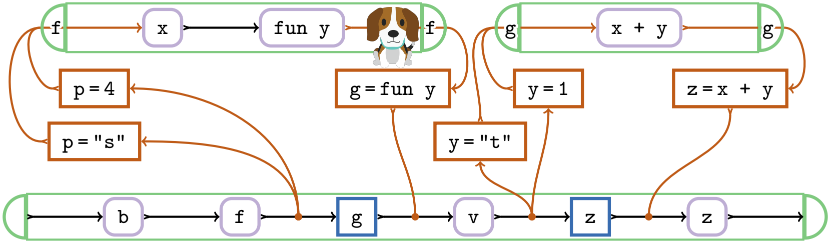

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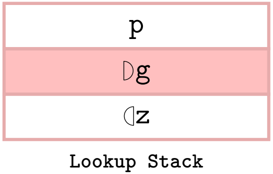

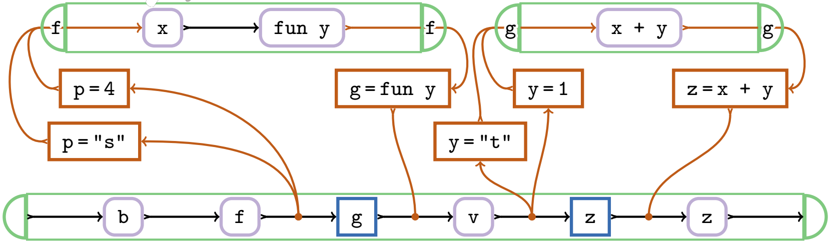

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```
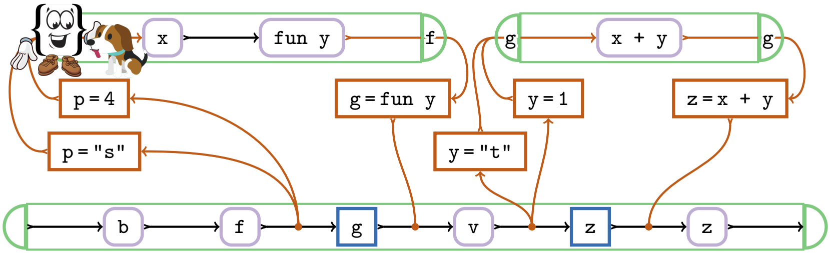

$\{p@[] \mapsto 4\}$
◗g
◖z

Lookup Stack



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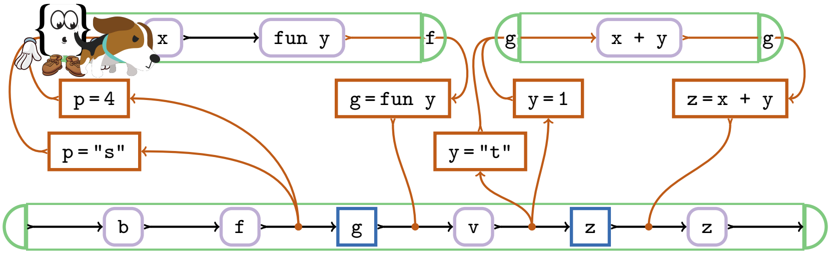
b
{p@[] $\mapsto$ 4}
◗g
◖z

Lookup Stack



DRSF and Variable Alignment

```
let b = coin_flip () in
let f = fun p ->
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      fun y -> x + y
in
let g = f (b?4:"s") in
let v = (b?1:"t") in
let z = g v in z
```
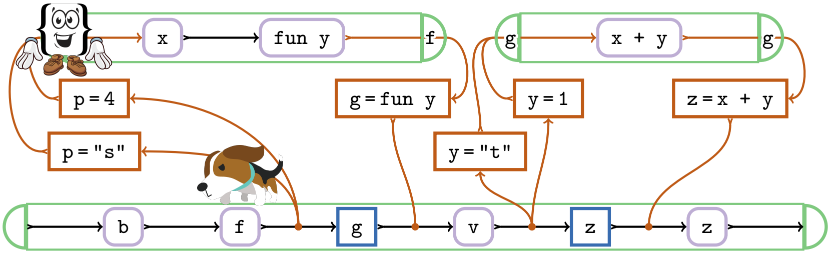

b
◖g
{p@[] $\mapsto$ 4}
◗g
◖z

Lookup Stack



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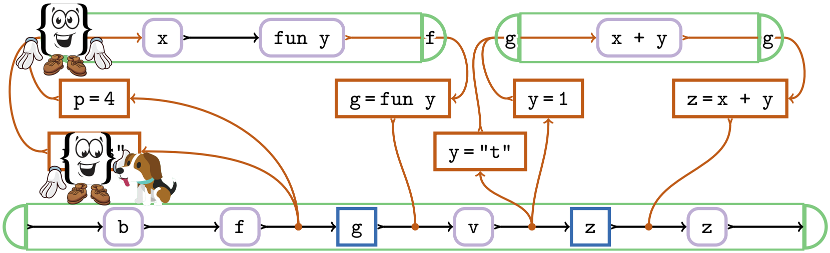
{b@[] ↦ true}
◖g
{p@[] ↦ 4}
◗g
◖z

Lookup Stack



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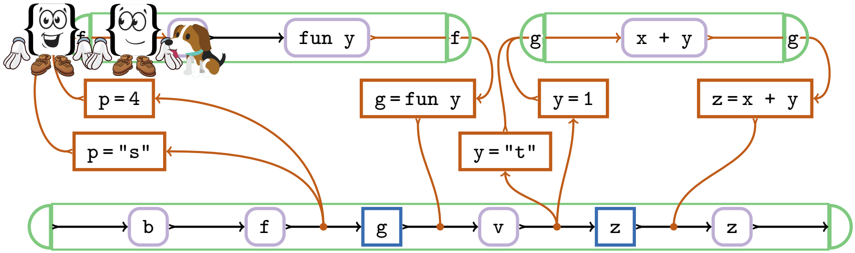
{b@[◖g] ↦ true}
{p@[] ↦ 4}
◗g
◖z

Lookup Stack



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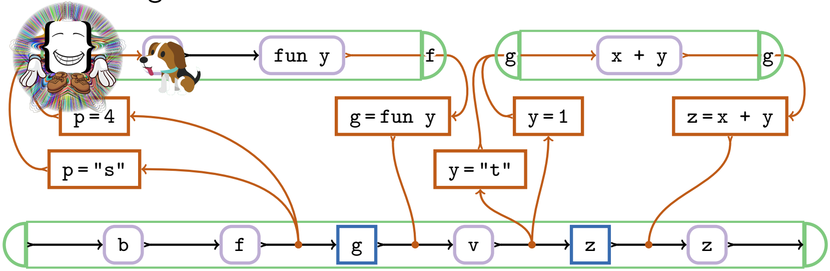
$\left\{ \begin{array}{c} \texttt{p@[]} \mapsto 4, \\ \texttt{b@[◖g]} \mapsto \texttt{true} \end{array} \right\}$
◗g
◖z

Lookup Stack



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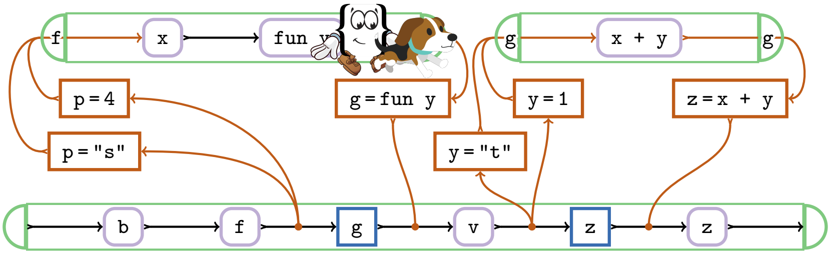
$$\left\{\begin{array}{l} \texttt{p}@[◗\texttt{g}] \mapsto 4, \\ \texttt{b}@[] \mapsto \texttt{true} \end{array}\right\}$$

◖z

Lookup Stack



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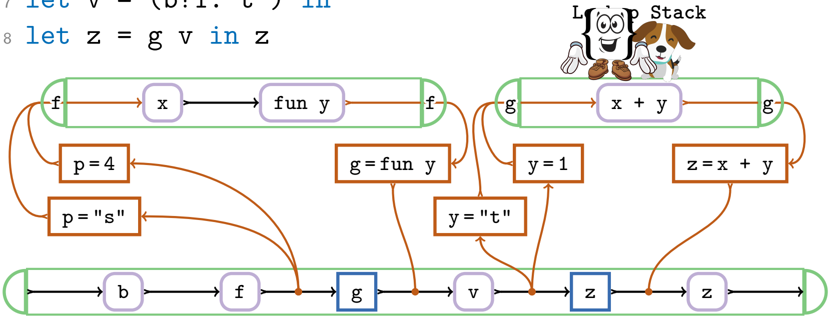

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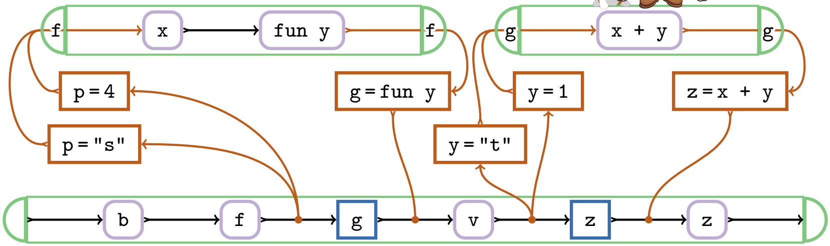

Merging Relative Store Fragments

$$\left\{\begin{matrix} \texttt{x@[]} \mapsto 4, \\ \texttt{b@[}\LEFTcircle\texttt{z]} \mapsto \texttt{true} \end{matrix}\right\} \oplus \left\{\begin{matrix} \texttt{y@[]} \mapsto 1, \\ \texttt{b@[}\LEFTcircle\texttt{z]} \mapsto \texttt{true} \end{matrix}\right\} =$$

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Polymorphism via Δ

```
let f = fun x -> x in
let a = 4 in
let b = f a in
let c = "s" in
let d = f c in
0
```

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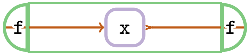

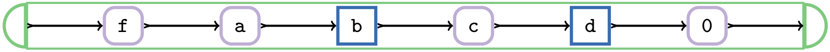

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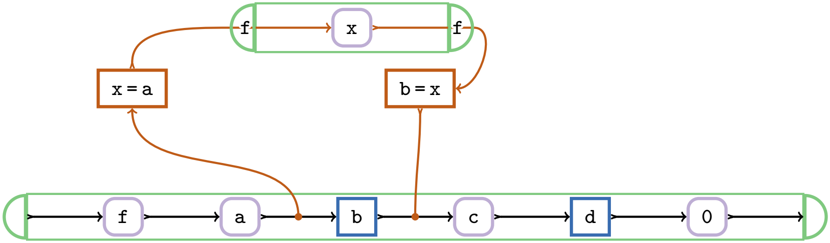

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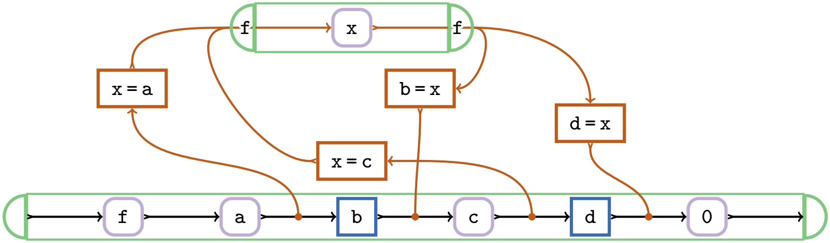

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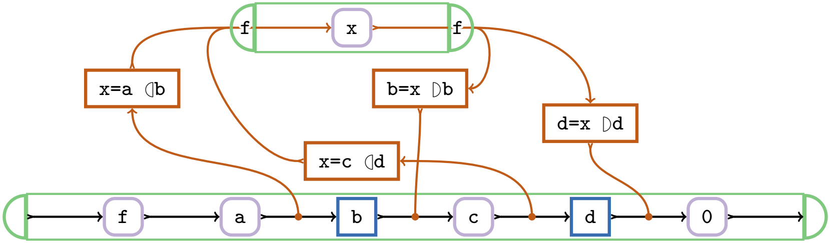

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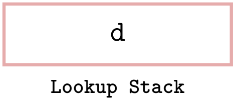

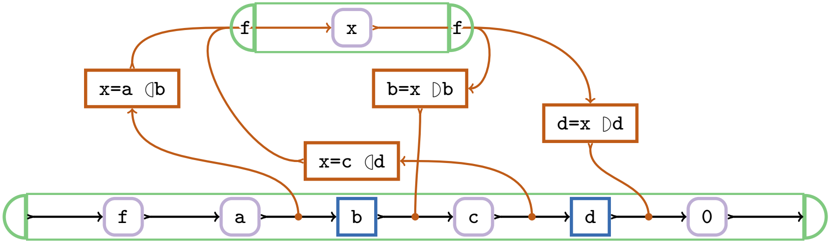

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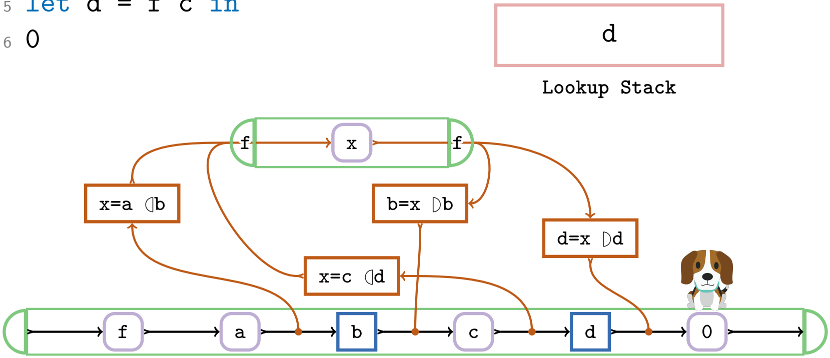

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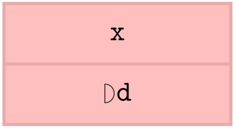
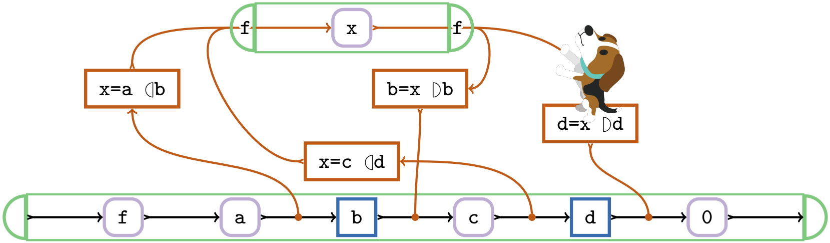


Lookup Stack



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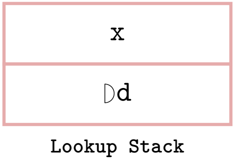

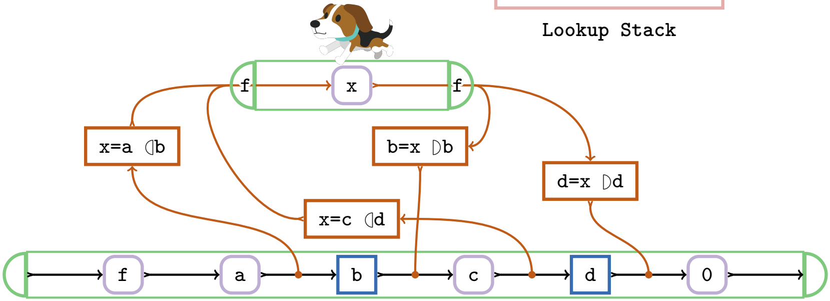

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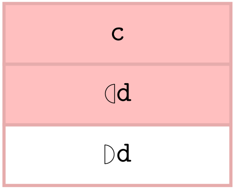

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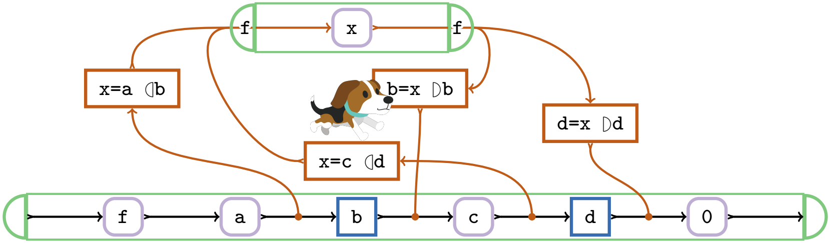

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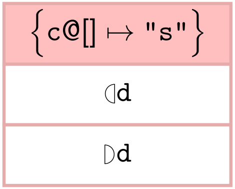

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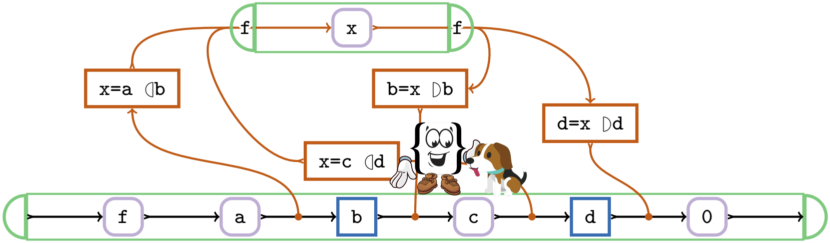

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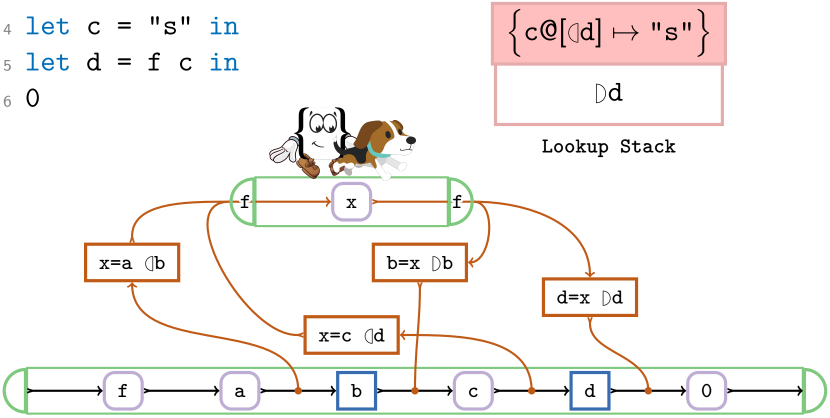

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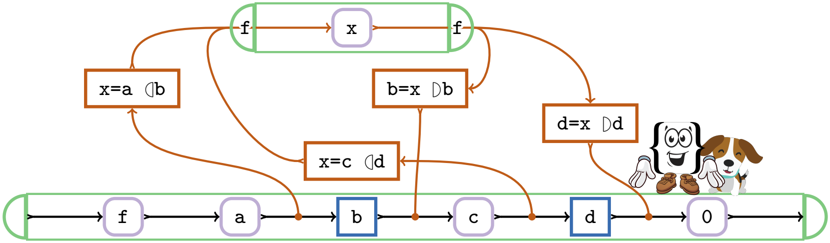
```

$\left\{ \texttt{c@[]} \mapsto \texttt{"s"} \right\}$

Lookup Stack



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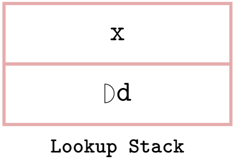

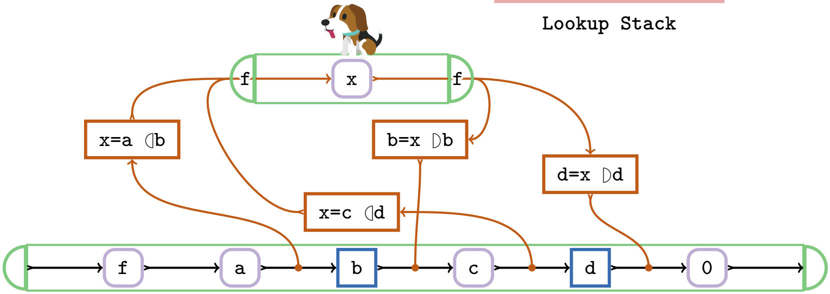

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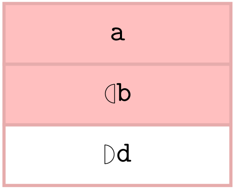
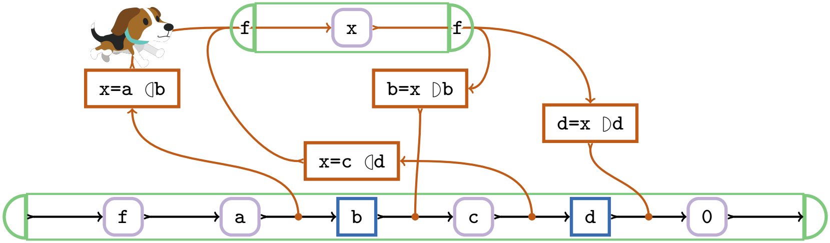


a
◖b
◗d

Lookup Stack



Polymorphism via Δ

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```
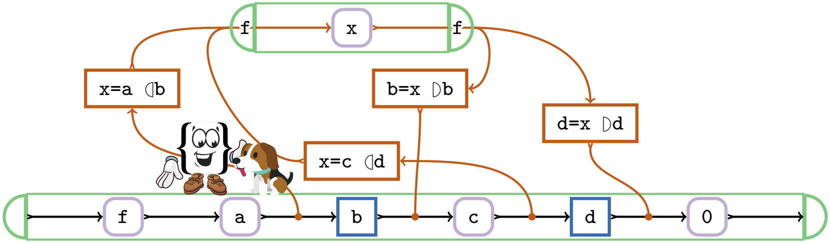

$\left\{ \texttt{c@[]} \mapsto 4 \right\}$
◖b
◗d

Lookup Stack



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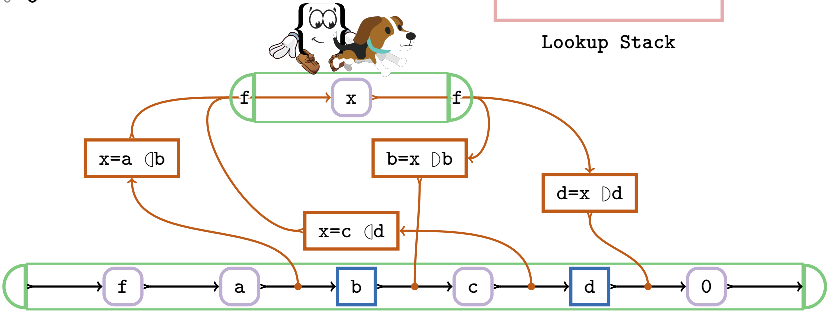
```

$\left\{ \mathtt{c}@[◖\mathtt{b}] \mapsto 4 \right\}$
◗d

Lookup Stack



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```
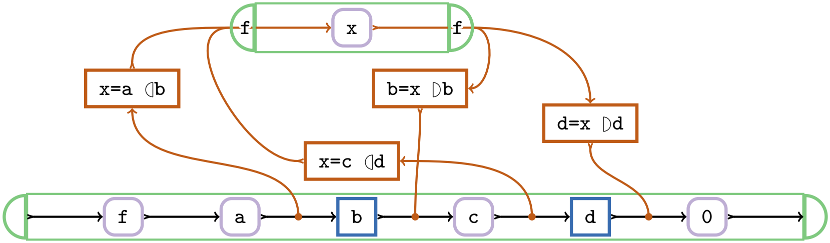

✗

Lookup Stack



Singleton Abstractions via Full Traces

- Traces Δ represent relative stack adjustments

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  - Flow-sensitivity
  - Path-sensitivity
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  - Non-local variable alignment

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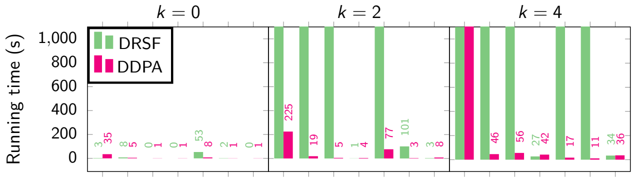

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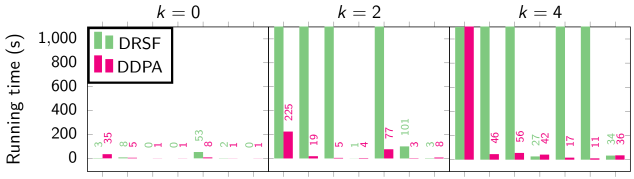

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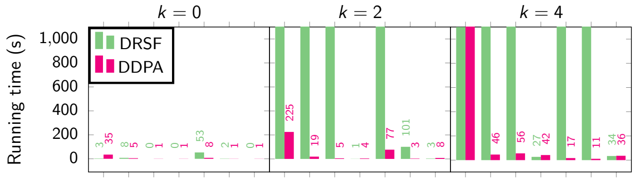

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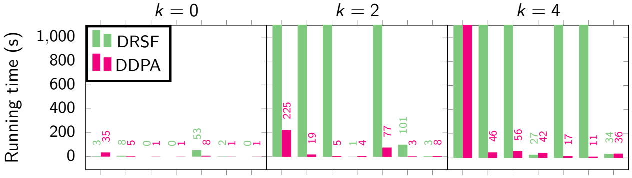

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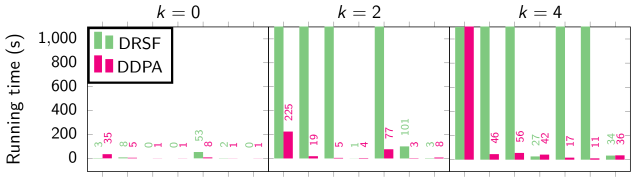

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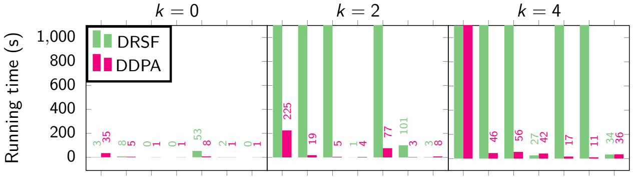

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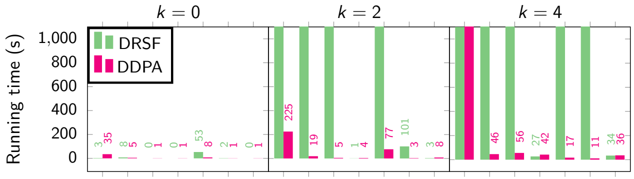

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Questions?